MANUAL OF PHYSICAL MEASUREMENTS

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A MANUAL

OF

PHYSICAL MEASUREMENTS

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PREFACE TO THE SIXTH EDITION

This manual is an outline of the laboratory experiments given in the courses in general physics at the University of Minnesota. The laboratory work in these courses supplements the lectures and recitations.

The experiments given in a junior course of one quarter in electricity measurements are included and are given under the title of Electricity II.

It is taken for granted that the student has acquired a general knowledge of a subject before it is considered in the laboratory. No attempt is made in the manual to achieve completeness in subject matter or explanation. The work is done under the guidance of an instructor who furnishes any additional information that may be required.

The student should feel that acceptable results depend upon his own ability to adjust the apparatus properly; and he alone should plan and execute the details of the experiments, subject, of course, to the criticism of the instructor.

We here wish to thank Professors J. W. Buchta, L. F. Miller, J. Valasek, and John H. Williams for valuable suggestions, criticisms, and assistance during the preparation of this manual.

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INTRODUCTION

Work in the physical laboratory brings the student into first-hand touch with physical principles and physical apparatus, and the impressions thus produced through the senses furnish a solid foundation for further study.

The close attention to every detail and the exercise of deliberate judgment which are required in every experiment if a worthy result is to be obtained tend to produce a habit which is of inestimable value. This training is acquired only when every effort is made to obtain the very best result that the time allowed for the experiment and the apparatus employed will permit.

Before observations in any experiment are begun, the theory of the experiment should be mastered as well as the functions of the various parts of the apparatus which is to be used. Without such study, necessary observations may be omitted or taken in an improper manner, and apparatus whose value depends upon its accuracy may be injured permanently because of a lack of knowledge of its delicate parts.

In a measurement of a physical quantity, one not only wishes to know the magnitude of the quantity but also desires to have some information regarding the accuracy with which the measurement has been made.

In counting the number of objects or parts of an object, one can obtain the exact number; but in measuring physical quantities, such as lengths, time intervals, etc., it is obvious that absolutely correct values cannot be obtained. The accuracy of any measurement of this character is limited by the inability of our senses to distinguish small differences in space and time, by imperfections in the measuring instruments, and by other factors.

The main sources of error and some useful methods of treating observations are here tabulated and then discussed:

KINDS OF ERROR AND TREATMENT OF ERRORS AND DATA

1. Determinate Errors
a. Instrumental errors
b. Errors due to change of condition
c. Personal errors

2. Errors of Chance
a. Estimated error in a single reading
b. Distribution of chance errors
c. Probable error in a single reading
d. Probable error in the average
e. Application
f. Experimental and observational errors

3. Probable Error in Computed Results
a. In addition and subtraction
b. In multiplication and division
c. In raising a quantity to any power

4. Rejection of Observations—Huge Errors

5. Best Use of Data

6. Graphical Representation of Results

7. Use of Logarithm Paper

1. Determinate Errors.—Determinate errors are fixed errors due to a more or less constant cause. Their cause or magnitude therefore can be found and the proper adjustment or correction made. These errors may be classified as follows:

a. Instrumental errors are errors which arise from a faulty construction or adjustment of an instrument, such as unequal balance arms, faulty scale graduations, etc., and all permanent changes in the instrument since it was standardized. It is often difficult to make these corrections, but usually they can be made.

b. Errors due to change of condition are those errors which arise from using an instrument under conditions other than those for which the graduations or constants are certified. Under this class of errors may be included the change in magnitude of the measuring unit due to changes in temperature, barometric pressure, etc. For example, the scale divisions, at the time the instrument is used are either shorter or longer than they were at the temperature at which the scale was standardized. The proper correction for such changed conditions can, in general, be made.

c. Personal errors arise from bias or tendencies of the observer. For example, one observer regularly records the occurrence of an event too soon, and another observer records it too late. The "personal equation" of an observer for correcting his observations can be determined experimentally.

2. Errors of Chance.—The errors of chance are chance errors (often called *accidental errors*) and are due to a variety of causes over which the observer has no control. They include errors in estimating to tenths of the smallest division or interval, to eye-strain, to sudden uncontrollable fluctuations in temperature, etc. The distribution of such errors has been shown to follow the definite "law of chance." By the use of this "law," an estimate of the magnitude of all such errors together can be made. This combined magnitude, however, is decreased by an increase in the technical skill and in the care taken by the observer. The methods for determining approximately the magnitudes of the errors of chance are as follows:

a. Estimated Error in a Single Reading.—By means of a scale, for example, a measurement can be made to within a fraction of the smallest scale division. The usual practice is to make the estimate to one-tenth of the smallest division if the divisions are large enough to make that possible. The readings, however, may or may not be correct to within one of these estimated parts. The digit in the reading which designates the estimated fraction of the smallest division is called the *last significant figure* in the reading. For example in the reading 86.37 cm, the magnitude of the last digit 7 is estimated, and therefore the digit 7 is the last significant figure of the observation and may be in error by one or more of the estimated parts. If the observer has reason to believe that his estimate is correct to within 0.1 of the smallest scale division, as is usually the case, his single reading is expressed by 86.37 ± 0.01 cm or by 86.37 ± 1 cm. By either form, the figures give the measured length to be between the limits 86.36 cm and 86.38 cm. If the observer has reason to believe that his measurement may be in error by 0.3 of the smallest division, he expresses his measurement by 86.37 ± 0.03 cm or by 86.37 ± 3 cm.

b. Distribution of Chance Errors.—If a very large number n of independent readings for the magnitude of a quantity is

taken with the same instrument, each individual reading differs from the average value of all the readings by some amount d. This difference d is called the *deviation* of that reading from the mean. Experiment shows that small deviations occur with greater frequency than the large ones and that the deviations of + and − sign are equal in number and similarly distributed as represented in the "normal" error curve A, Fig. 1. Here the

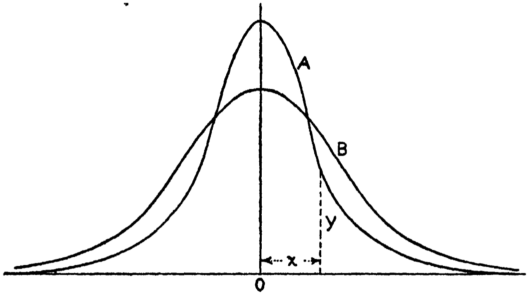


FIG. 1.

ordinates represent the number of deviations and the abscissas their magnitudes. For example, the ordinate y represents the number of deviations having the + magnitude x. The curve B represents the distribution of the deviations for the same number of readings taken with the same instrument by a less experienced observer or by the same observer when he is using another measuring instrument. These limiting curves represent the law of chance and are expressed by the equation

$$y = Ae^{-kx^2} \tag{1}$$

where A and k are constants, e is the base of the Naperian system of logarithms, and y is the number of deviations of the magnitude $\pm x$.

c. Probable Error in a Single Reading.—The probable error (p.e.) in a single reading is defined as the "error" or deviation which has a magnitude such that in a large number of observations there are as many deviations of smaller as there are of larger magnitude. In other words, a single observation has a 50–50 chance of not differing from the average of the many such observations by more than the magnitude of the probable error (p.e.). The probable error in a single observation is shown, by means of the normal error curve, Fig. 1, to be

$$\text{p.e.} = 0.67\sqrt{\frac{\Sigma d^2}{n(n-1)}} \cong 0.85\frac{\Sigma d}{\sqrt{n(n-1)}}, \tag{2}$$

where Σd is the arithmetical sum of all the deviations (regardless of sign) and n is the number of observations.

d. Probable Error in the Average.—The probable error (P.E.) in the average of n readings can be shown, in a similar manner, to be

$$\text{P.E.} = \frac{\text{p.e.}}{\sqrt{n}} \cong 0.85\frac{\Sigma d}{n\sqrt{n-1}} = K\Sigma d. \tag{3}$$

The following table gives the magnitudes of K for each of n observations up to ten.

When n =	2,	3,	4,	5,	6,	7,	8,	9,	10,
K =	.425,	.200,	.123,	.085,	.063,	.049,	.040,	.033,	.028.

e. Application.—In applying the foregoing equations, the student must remember that these equations refer to chance errors only, that they are known to give a reliable estimate of the probable error when a large number of observations on a single measurable quantity is taken in which the observed values of the last significant digit differ, and that they give a less accurate estimate of the probable error when the number of repeated observations is small, as is necessarily the case in most laboratory experiments.

Whenever identical or practically identical readings for the last digit are obtained, the theory of chance does not apply. In such a case, the probable error may be taken arbitrarily to be 0.5 of the estimated part of the smallest division or interval unless some other value appears to be more reasonable.

Suppose the following to be the n repeated observations on a length and their individual deviations d from the average:

Length, cm	d, cm
2.53	0.008
2.55	0.012
2.54	0.002
2.54	0.002
2.53	0.008
Average = 2.538	Σd = 0.032

The probable error in a single reading, since $n = 5$ and $\Sigma d = 0.032$, is, from Eq. (2),

$$\text{p.e.} = 0.85\frac{\Sigma d}{\sqrt{n(n-1)}} = 0.85\frac{0.032}{\sqrt{5(5-1)}} = 0.006 \text{ cm.}$$

The probable error in the average, from Eq. (3), is

$$\text{P.E.} = 0.85\frac{\Sigma d}{n\sqrt{n-1}} = 0.85\frac{0.032}{5\sqrt{5-1}} = 0.003 \text{ cm.}$$

All figures to the right of the first one affected by the calculated probable error have no significance and should be omitted. Hence if the average of a number of readings is 10.5824 and the P.E. is 0.01, the average of the readings is expressed by 10.58 ± 0.01 or by 10.58 ± 1. This means that, as far as the chance errors are concerned, the chance is one out of two that the number 10.58 is not in error by more than 0.01. If a calculated result is expressed by the number 10582465 and the P.E. affects the digit 8 by ± 2, the facts are expressed by 10580000 ± 20000, or better by $(10.58 \pm 2) \times 10^6$.

f. Experimental and Observational Errors.—The terms *experimental* and *observational error* differ in meaning from the term *probable error.* The *experimental error* includes errors due to imperfect construction or adjustment of an instrument and all other uncorrected sources of error, as well as the chance errors. An estimate of the magnitude of an experimental error can be found by measuring the quantity several times with different adjustments and under different experimental conditions as well as with different observers and different instruments. This can rarely be done in an elementary laboratory.

When two independent determinations are said to *check within the probable error,* the meaning is clear and obvious. When one states that the results *check within the experimental error,* one usually means that they do not differ by more than what experience shows can be attributed to the various possible sources of error. The meaning is necessarily somewhat indefinite. However, one should feel dissatisfied if the results do not check within three times the probable error P.E.

The term *observational error* is usually used synonomously with experimental error. It, however, tends to imply that the chief

part of the error in a given determination may be due to the limitation imposed by the method of taking or using the observations.

3. Probable Error in Computed Results.—The contribution of the probable errors of several measured quantities to the probable error in the computed result may be obtained with fair approximation as follows:

a. In addition or subtraction retain as the probable error in the result the *numerically largest* probable error occurring in any one of the contributing quantities. For example

$$\begin{array}{r} 45.23\pm 8\\ 25.6\pm 2\\ 2.588\pm 1\\ \hline 73.4\pm 2 \end{array} \qquad \begin{array}{r} 45.23\pm 8\\ 25.6\pm 2\\ \hline 19.6\pm 2. \end{array}$$

b. In multiplication or division the probable numerical error in the result is obtained from the probable *percentage* error, which is taken to be the same as the probable percentage error of the least accurate factor. For example

$$(4.05 \pm 2) \times (60.41 \pm 3) = 245. \pm 1$$
$$(60.41 \pm 3) \div (4.05 \pm 2) = 14.91 \pm 7.$$

The probable error in per cent of the factors is largest in the factor 4.05 and, within significant limits, is 0.5 per cent of that factor. In the product or quotient, then, the probable error is 0.5 per cent of the whole quantity. This 0.5 per cent changed to numerical magnitudes gives the product a probable error of ± 1 in the units place and the quotient a probable error of ± 7 in the second decimal place.

c. In raising a quantity to any power, the probable percentage error in the result is obtained by multiplying the probable percentage error in the original quantity by the power to which the quantity is raised.

Thus

$$(a \pm e\%)^n = a^n \pm ne\%. \tag{4}$$

For example, if the exponent $n = 2$, the probable percentage error in the result is twice that in the observed quantity a, and, if the exponent is ½, the percentage error is one-half that in a.

The corrections for determinate errors are made whenever they are larger or comparable to the probable (chance) error P.E. Of these the *instrumental error* is usually corrected by means of correction tables or correction curves, and in many cases also by planned repetition of observations; the correction for *errors due to change of condition* is made by use of tables of temperature coefficients, etc.; the *personal error* is usually ignored in an elementary laboratory, because it is usually smaller there than the probable (chance) error and because of the difficulty and time involved in determining the personal equation. After the corrections for determinate errors have been made, the probable error P.E. gives a reasonable assurance of the degree of accuracy with which the magnitude of the quantity under observation has been determined. If the corrections for determinate errors have not been made, the probable error still gives valuable information and should be recorded even though the determinate errors may be larger than the chance errors.

4. Rejection of Observations—Huge Errors.—Individual observations which clearly indicate some mistake are discarded. But in a series of independent observations on a single quantity there sometimes appears a reading which varies greatly from the average although the observer cannot ascribe the large deviation from the mean to any mistake. One cannot reject such a reading from the group without applying an accepted criterion. When a single reading deviates from the average by more than 4.9 times the probable error of a single observation, Eq. (2), that reading should be rejected. The law of chance gives the chance to be only one in one thousand that an observed reading deviates that much from the average.

5. Best Use of Data.—When for example, a length is measured, the readings are taken for both ends of the measured object. The difference in the readings gives one measurement of the length. In repeating the measurement, the scale is shifted slightly so that the fractions of the smallest divisions to be estimated are, in general, not the same in the several repetitions. Such readings and measurements are said to be independent readings and measurements. The average of the several independent determinations of the length is the most probable magnitude of the length, with the chances being one out of two

that the chance error is not greater than that given by the probable error as treated in the foregoing sections. Whenever the determinate corrections for temperature and the calibration of the scale are large enough to be comparable to the probable error, these corrections must be made before the measurement is complete.

It often happens, however, that readings are consecutive, such as those of the time of day taken when a mark on a revolving wheel crosses an index line in the measurement of the period of revolution. Let a, b, c, d, e, f, g, h represent these observations. If the period t of one revolution is calculated from

$$t = \frac{(b-a)+(c-b)+(d-c)+(e-d)+(f-e)+(g-f)+(h-g)}{7}, \tag{5}$$

it is evident that the computation, in reality, is made by use of the equation

$$t = \frac{h-a}{7} \tag{6}$$

and is based on only the two end readings a and h.

A different arrangement of the readings, however, makes the period

$$t = \frac{\dfrac{(e-a)}{4}+\dfrac{(f-b)}{4}+\dfrac{(g-c)}{4}+\dfrac{(h-d)}{4}}{4} = \frac{(e+f+g+h)-(a+b+c+d)}{16}, \tag{7}$$

where all the readings contribute toward the measurement and thereby increase the accuracy of the determination. It is often sufficient, when a large number of readings is taken, to use only a few of the end readings in this manner.

6. Graphical Representation of Results.—The relation between two varying quantities may be more clearly shown by plotting them on coordinate paper, one quantity being represented by distances parallel to the X axis and the other by distances parallel to the Y axis. The choice of units to be used for laying off these quantities is arbitrary and need not be the same for the two axes. However, in order to obtain the most complete representation,

the unit should be so chosen, where practicable, that the smallest division on the coordinate paper represents a unit of the last significant figure of the measurement. The unit selected should be recorded along each of the axes.

Convenient equally spaced division lines should be numbered in the units of the quantities to be plotted, as shown in Fig. 2.

The individual readings are not recorded on the coordinate paper, but are tabulated and included in the report of the experi-

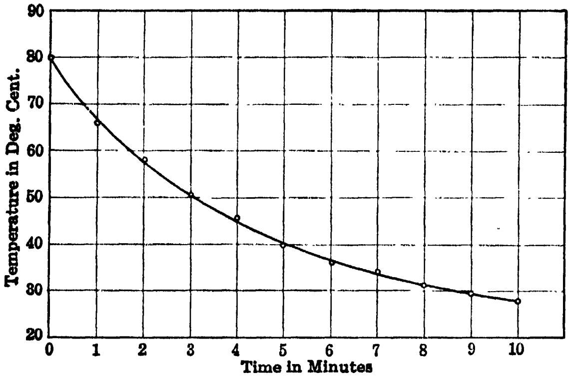


FIG. 2.

ment. The observed points, however, are marked clearly by crosses or circles and a smooth line is drawn through these as shown.

7. Use of Logarithm Paper.—The law governing a phenomenon may often be determined or checked most readily and critically if the data can be incorporated in an equation of the linear form. This may frequently be accomplished by the use of logarithmic relations. For example, let the relation between the distance S traveled and the time t in the case of a falling body be expressed by

$$S = At^n, \tag{8}$$

where A is a constant. It is desired to determine experimentally the magnitude of A as well as that of the exponent n. Passing to logarithms, the Eq. (8) becomes

$$\log S = \log A + n \log t = K + n \log t, \tag{9}$$

where $K = \log A$ = a constant. This gives a linear relationship between $\log S$ and $\log t$.

It is possible to rule paper in a manner such that the abscissas and ordinates represent the logarithms of the numbers given on the coordinate axes. Such ruled paper, called *log-log paper*, is available.

Suppose the following observations have been taken in the case of a body falling freely from rest:

S	t
1	0.250
2	0.355
3	0.433
4	0.500
5	0.557
6	0.608
7	0.661
8	0.707
9	0.750
10	0.790

Plotting these observations on the log-log paper as shown in Fig. 3, the plotted points fall in a straight line whose intercept on

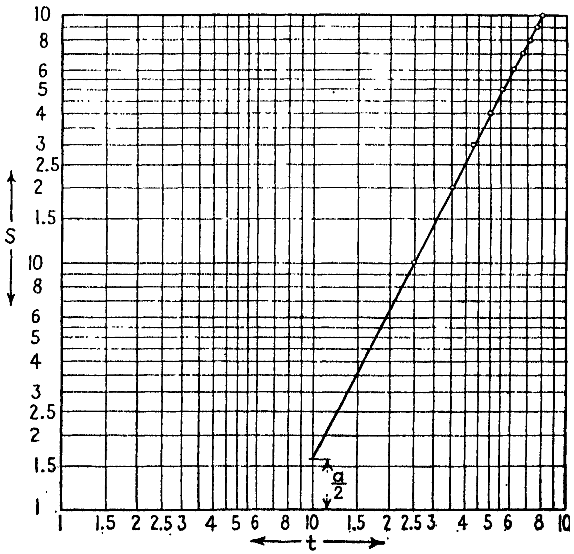


Fig. 3.

the axis of ordinates is the constant $K = \log A$. The intercept then, because of the nature of the ruling, gives directly, without any transformation, the magnitude of the constant A. This magnitude is found to be 16, or one-half that of the acceleration due to gravity. The tangent of the angle of slope gives the magnitude of n, which the plot shows to be 2 within the limits of accuracy of the graphical representation of the slope. The plot then proves that within the limits of accuracy of the graphical representation the space traveled by a freely falling body is

$$S = At^n = 16t^2 = \tfrac{1}{2} at^2. \tag{10}$$

CONTENTS

APPENDIX

TABLES

CHAPTER I

MECHANICS

EXPERIMENT 1

TO MEASURE A LENGTH IN CENTIMETERS AND IN INCHES DIRECTLY BY MEANS OF A SCALE

Apparatus.—A metric scale; an English scale; a cylinder the length of which is desired.

Method and Manipulation.—Place the cylinder against the side of the scale so that the two are parallel, and so that the scale extends beyond the ends of the cylinder. Do not try to adjust until a line on the scale is even with an end of the cylinder. The distance that the cylinder extends into a division must be determined by estimating to tenths. A division is the space between the centers of two adjacent lines on the scale. Divide this distance mentally into halves and the half into which the cylinder or object extends into five parts, each of which then is a tenth of the whole division.

The difference between the readings obtained for the two ends, reckoned in each case from the zero end of the scale, gives the length of the cylinder with an accuracy depending upon the accuracy of each reading.

In repeating observations, place the cylinder each time opposite a different portion of the scale.

Obtain in this manner several values for the length in centimeters by using the scale on which the smallest division is a millimeter and likewise the scale on which the smallest division is a sixteenth of an inch. Solve for the number of centimeters in an inch, and find the last significant figure in result by applying the rules given in the introductory chapter.

Sources of Error.—The scale and object may not be parallel.

The point read on the scale may not be exactly opposite the end of the object. This error is generally due to parallax; *i.e.*, the eye is not in a plane that is perpendicular to the scale at the

point where the reading is taken. Parallax is a frequent source of error which must be constantly guarded against.

EXPERIMENT 2

TO DETERMINE THE VOLUME OF A CYLINDER

Method.—The length and diameter of the cylinder are measured, and from these quantities the volume is computed.

Apparatus.—Vernier caliper; micrometer caliper with holder; magnifying glass; the cylinder used in Exp. 1.

I. TO MEASURE THE LENGTH OF THE CYLINDER BY MEANS OF A VERNIER CALIPER

The *vernier caliper* (Fig. 1) consists of a graduated limb *a*, with a fixed jaw perpendicular to it, and a second movable jaw *c*,

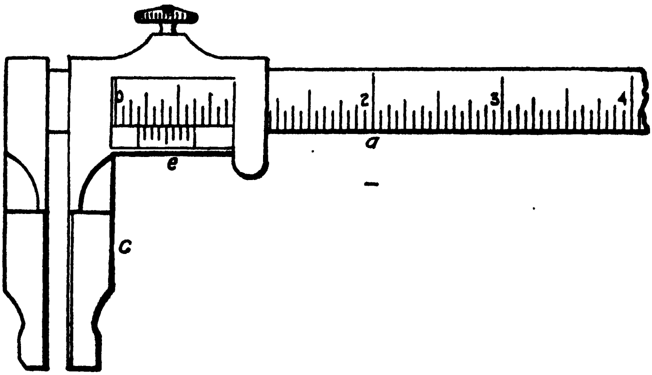


FIG. 1.

parallel to the first and provided with a series of lines *e*, called a vernier. The movable jaw may be placed anywhere along the limb *a*. The clamping screw is rarely used and has been removed from the calipers supplied. Since the initial line on the vernier (at the left in the figure) coincides with the zero line on the scale of the limb when the jaws are together, the position of this line at any time gives on the scale the separation of the jaws. When the initial line falls between two divisions on the scale, the fraction of a division is not estimated by the eye but is measured by means of the vernier.

The Vernier.

Let S_1 = the length of a division on the scale,
S_2 = the length of a division on the vernier,
n = the number of divisions on the vernier.

The number of divisions n on the vernier equals in length $(n - 1)$ divisions on the scale of the limb.

In Fig. 2 the initial line of the vernier coincides with line 4 on the scale. Line 1 is, then, the distance $(S_1 - S_2)$ from line a; line 2 is the distance $2(S_1 - S_2)$ from b; etc. If, now, the vernier is moved until line 1 coincides with line a, the initial line of the vernier is the distance $(S_1 - S_2)$ from line 4. In this case the reading of the initial line is $4 + (S_1 - S_2)$. If line 2 coincides with b, the initial line is $2(S_1 - S_2)$ from 4; etc. It is seen that the fraction of a division the initial line is away from

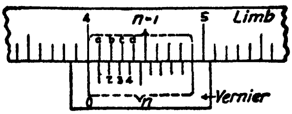


Fig. 2.

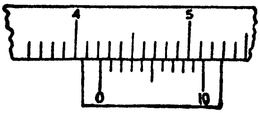


Fig. 3.

4 is $(S_1 - S_2)$ multiplied by the number of the vernier line that exactly coincides with a line on the scale. The quantity $(S_1 - S_2)$ is called the *least count* of the vernier. The value of $(S_1 - S_2)$ is determined in the following manner:

$nS_2 = S_1(n - 1) = S_1 n - S_1$ (by construction of the vernier).

$$\therefore\ S_1 n - S_2 n = S_1.$$

$$\therefore\ (S_1 - S_2) = \frac{S_1}{n}.$$

The least count thus equals the length of a division of the scale divided by the number of divisions on the vernier. The reading of the vernier in Fig. 3 is 4.23.

(On some verniers n divisions do not equal $(n - 1)$ divisions on the scale but some other whole number. The value of the least count in these cases is determined by similar reasoning.)

When no line on the vernier exactly coincides with a line on the scale, *i.e.*, when two adjacent lines of the vernier are within a division on the scale, a fraction of the least count must be estimated.

When the vernier lines are at equal distances from the scale lines, as shown in Fig. *a*, the fraction of the least count is 0.5. In Fig. *b*, when one line appears twice as far from a scale line as the other, the reading is 0.3 of the least count; in Fig. *c* it is 0.2.

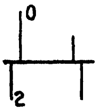


FIG. *a*.

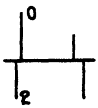


FIG. *b*.

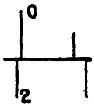


FIG. *c*.

Manipulation.—Find the least count of each vernier on the caliper. Bring the jaws together, and note the reading of the initial line when the jaws touch.

The difference in the reading when the jaws are together and the reading when the object is between them is the length of the object. Obtain the length of the cylinder in terms of each scale on the caliper. Reduce the length in inches (there will be as many of these as there are inch-reading verniers) to centimeters, and include them in the final average.

Sources of Error.—1. Jaws may not be parallel.

2. The object measured may not be parallel to the scale.

3. Too great pressure may bend jaws or change dimensions of the object measured.

4. Dust between the jaws of the calipers may cause the zero readings to be too great.

II. TO MEASURE THE DIAMETER OF THE CYLINDER BY MEANS OF A MICROMETER CALIPER

The *micrometer caliper* (Fig. 4) consists of a uniform screw moving in a fixed nut.

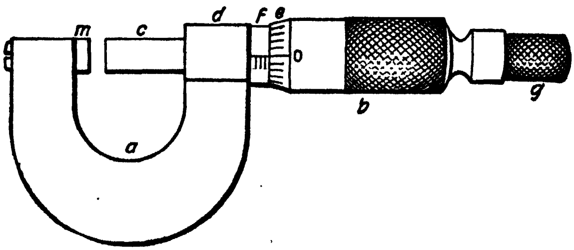


FIG. 4.

The nut *d* has a scale *f* and carries the arm *a*. The head of the screw *b*, with its divisions at *e*, is called the micrometer head. The end of the screw *c* can be moved against any object placed in the opening *m*.

Examine the caliper, and determine the following quantities:

1. Length of a division on the linear scale on the nut.
2. The pitch of the screw.
3. Number of divisions on the micrometer head.
4. The distance advanced by the screw when it is turned through one division on the micrometer head.
5. The fraction of a millimeter that can be obtained by estimating.

Manipulation.—To facilitate the taking of readings, place the caliper in the holder. Determine the zero reading of the micrometer. Do not force the screw. Stop when the head first touches the object. A friction head *g*, if provided, insures the application of the proper force. Turn until the friction head begins to move or, if there is a ratchet attachment, until it clicks.

Hold the cylinder loosely between the jaws of the caliper. This practically insures getting the cylinder perpendicular to the jaws.

Measure the diameter at several points along the length of the cylinder, and determine the average diameter.

Sources of Error.—1. The pitch of the screw may not be known accurately and may vary in different parts.

2. Change of temperature of the caliper if held by hand may change the zero reading during the experiment.

III. Compute the Volume of the Cylinder in Cubic Centimeters from the Average Values of the Length and Diameter Obtained Above

If S is the length, and $2R$ is the diameter of a right circular cylinder, its volume V is given by the expression

$$V = \pi R^2 S.$$

From this evaluate the volume, and determine the last significant figure in the result by applying the rules given in the introductory chapter.

EXPERIMENT 3

TO DETERMINE THE RADIUS OF CURVATURE OF A SPHERICAL SURFACE BY MEANS OF A SPHEROMETER

Apparatus.—A spherometer; a piece of plate glass; a sheet of paper; dividers; metric scale; a spherical surface.

The *spherometer* consists of a screw with a micrometer head. This screw turns in a nut which is supported by three legs at equal distances from each other. The screw and legs end in blunt points. The screw is in line with the center of the triangle formed by the points of the legs. A vertical scale attached to the nut is used to determine the number of whole revolutions of the micrometer head.

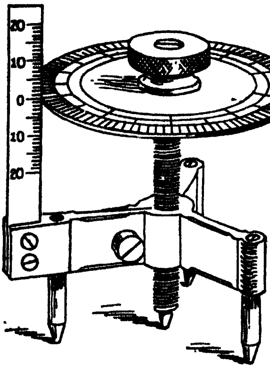


Fig. 5.

Determine the following quantities:

1. Length of a division on the vertical scale.
2. Pitch of the screw.
3. Value of a division on the micrometer head.

Method and Manipulation.—The spherometer is placed on the plate glass surface, and the micrometer head is turned until the screw just touches the plate and is thus in the same plane as the ends of the legs. To determine this condition, press very lightly on the nut with two fingers, one over each of two legs, and tap with the other hand over the third leg. If the spherometer hobbles, the screw is too low; and if it does not, it is likely to be too high. Find the point where the spherometer will not hobble but will do so if the micrometer head is turned through a tenth of a division. The reading of the scale and micrometer when the end of the screw and the ends of the three legs are in the same plane gives the *zero reading*.

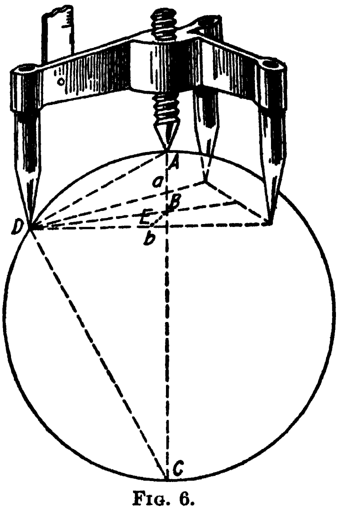


Fig. 6.

The spherometer is then placed on the spherical surface. The micrometer head is again turned until the four points touch the

surface. The scale and micrometer are read, and the distance d (Fig. 6) through which the screw has moved from its zero position is determined.

Turn the screw until the four points are again in the same plane Place the instrument on a piece of paper, and press lightly until the four points make small dents in the paper. By aid of dividers and scale, the average distance DB between the legs and the point of the screw is determined.

The radius of curvature is calculated from the distances d and DB, using the expression

$$R = \frac{1}{2}\left(\frac{\overline{DB}^2}{d} + d\right).$$

Suggestion.—The vertical scale usually has the zero mark at the center. This is liable to confuse beginners. It is better to call the highest mark on the scale zero.

EXPERIMENT 4

TO DETERMINE THE MASS OF A BODY

Apparatus.—The analytical balance.

The arrangement of parts is as shown in Fig. 7. The beam is usually supported by an agate knife-edge on an agate surface. It can be raised by means of a milled head in front of the case so that the knife-edge is not in contact with the supporting surface.

There are two adjustments that should be understood. The sensitiveness of the balance is dependent upon the distance between the knife-edge and the center of gravity of the beam. The shorter this distance is the greater the sensitiveness. This distance can be changed by raising or lowering the nut that is on the vertical screw above the beam.

Attached to the beam is a horizontal screw with a movable nut. By moving this nut, the beam may be adjusted so that the pointer comes to rest at the center of the lower scale.

On the beam, or attached to it, is a scale on which the distance between the central knife-edge and the edge supporting the scale pan is usually divided into 10 major parts. Smaller divisions, if present, are usually not used. Over this scale hangs a rider which can be placed at any point on the scale. This rider has a mass of 10 mg and is, therefore, when placed at the first division on the

scale, equivalent to a mass of 1 mg placed in the scale pan; when at the second, to 2 mg; etc. Brass weights are used for denominations ranging from the largest down to 1-g. For fractions of a

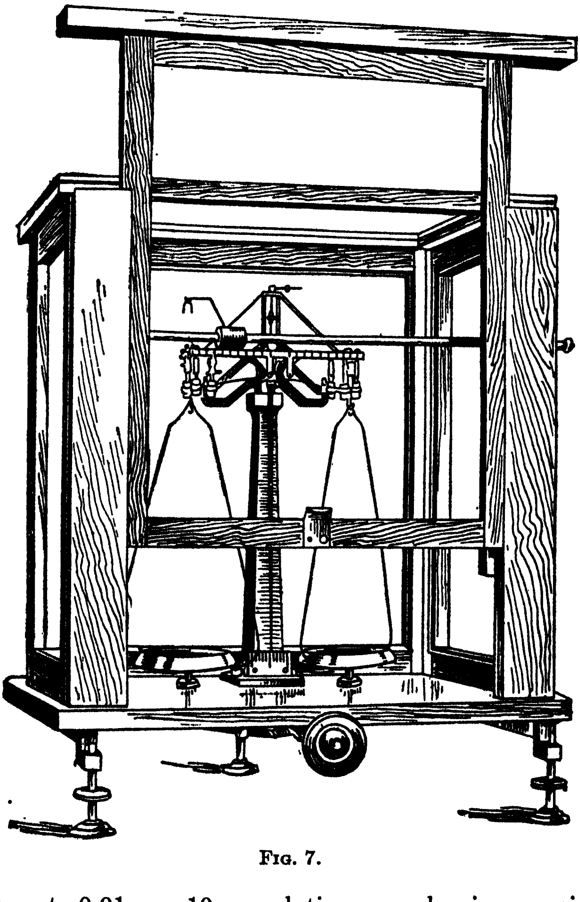


FIG. 7.

gram down to 0.01-g or 10 mg, platinum or aluminum weights are used. Less than 10 mg is obtained by means of the rider.

Method and Manipulation.—Release the beam carefully, and, if it is at rest, set it in vibration by raising and again releasing. Note at what place the pointer turns, counting the divisions from the left end of the scale and estimating to tenths of a division. Take five successive turning points. Call S_1, S_3, S_5 the readings

on one side and S_2, S_4, the readings on the other. Then the reading on the scale at which the pointer would stop is

$$Z = \frac{\frac{S_2 + S_4}{2} + \frac{S_1 + S_3 + S_5}{3}}{2}.$$

This is called the *zero point*. It is necessary to take one more reading on one side because of the decrease in the vibrations due to friction.

Raise the beam; place the body, the mass of which is desired, in the *left* pan; and add weights to the right until the pointer swings within the scale. Add weights only when the beam is raised, and try the effect of each addition by very carefully lowering the beam and noting which way the pointer begins to move. Take readings for turning points and average as in the case above. This gives the *point of rest* (R). If the point of rest happens to be the same as the zero point, the mass of the weights is equivalent to that of the body. If, however, the point of rest is less than the zero point, the mass of weights is too large; and if the point of rest is larger, the mass of the weights added is too small. In the last two cases we must determine what mass would have to be subtracted or added in order to bring the pointer to the zero reading. In order to do this we must know what is called the *sensibility* of the balance, *i.e.*, the deflection of the pointer that 1 mg will produce. To obtain this, change the weights by 1 mg, by means of the rider, and determine, in the same manner as above, another point of rest which we shall call the *sensibility point* (S). The difference between the *point of rest* and *sensibility point* gives the *sensibility*.

The mass to be added or subtracted is numerically equal to the difference between the zero point and the point of rest, divided by the sensibility.

Suggestions.—In getting the sensibility, add 1 mg if the point of rest is larger than the zero point, and subtract if it is less.

Always use the largest weights permissible. This facilitates the weighing, as it reduces the number of separate weights needed and reserves the smaller weights for the final adjustment.

The method described above is usually spoken of as the *method of vibrations*.

Errors and Precautions.—Before changing the weight, and when through weighing, always raise the beam of the balance. This should be done when the pointer is at the center of the scale.

Since air currents influence the swing of the beam, readings must not be taken immediately after closing the case of the balance.

Take the reading for the turning point when the eye, the pointer, and its image in the mirror back of the scale are in the same line. This avoids the error due to parallax.

Always handle the weights with the forceps. *Never* touch them with the fingers.

When through with a weight, *always* return it to the proper place in the case.

As the zero point is affected by temperature changes and slight shifting of the parts of the balance, a zero point must be determined after the load has been removed. The average of the first and last zeros is used in the interpolation.

EXPERIMENT 5

RATIO OF THE LENGTHS OF THE BALANCE ARMS

Method.—Place a body having a mass of about 50-g in the left pan, and obtain the apparent mass, using the method of vibrations. Then obtain the apparent mass when the body is in the right pan.

If m = the mass of the body,
m_1 = the apparent mass of the body when in the left pan,
m_2 = the apparent mass of the body when in the right pan,
S_2 = the length of the right arm,
S_1 = the length of the left arm,

then
$$mS_1 = m_1S_2,$$
and
$$mS_2 = m_2S_1,$$
$$\therefore \frac{S_2}{S_1} = \sqrt{\frac{m_2}{m_1}}.$$

Remarks.—If the left pan is used for the body, the apparent mass must be multiplied by the ratio S_2/S_1 in order to obtain the correct mass, since $m = m_1\frac{S_2}{S_1}$.

If the ratio is not known, the correct mass can be obtained by resorting to double weighing, as above, and taking the square root of the product of the apparent masses, *i.e.*, $m = \sqrt{m_1 m_2}$.

To Obtain the Mass in Vacuum.—In air the body and the weights are lighter by the weight of the mass of the air displaced by each.

If M = the mass of the body *in vacuo*,
m = the apparent mass of the body in air,
ρ_1 = the density of the air (see Table 19),
ρ_2 = the density of the weights,
ρ = the density of the body,

then

$$M = m\left(1 + \frac{\rho_1}{\rho} - \frac{\rho_1}{\rho_2}\right).$$

EXPERIMENT 6

A STUDY OF ACCELERATED MOTION

Apparatus.—An inclined metal track about 10 ft long, between the rails of which is placed an insulated metal strip, of length equal to the track, and over which is a paper ribbon; a car, mounted on ball bearings, from which a metal point projects toward the paper ribbon; an induction coil or a high-tension transformer (ratio 1 to 10) the primary of which is closed every half second by means of a relay in the clock circuit, the secondary being connected to the track and to the metal strip. Closing the primary releases the car by attracting the armature of an electromagnet and causes an electric discharge to take place each half second thereafter between the point projecting from the car and the metal strip beneath it, thus puncturing the paper ribbon and marking the position of the car, at successive half seconds.

Method and Manipulation.—Attach the car at the upper end of the track, and close the primary circuit. As the car descends, the punctures through the paper ribbon mark the positions of the car at each instant.

By means of a meter scale or steel tape obtain the readings in centimeters for each puncture by placing the zero of the tape at an arbitrary point preceding the first puncture.

1. Plot the total distances from the first puncture and the corresponding times on log-log paper, and from the slope determine the exponent of the time; from the intercept determine the value of the acceleration.

2. Plot, on the usual coordinate paper, the total distances as abscissas and the first, second, and third powers of the corresponding times as ordinates. A straight line means that the distances S are proportional to t^x, the corresponding ordinates.

From the tangent of the angle of slope of the straight line obtained above compute the value of a, the acceleration. Also,

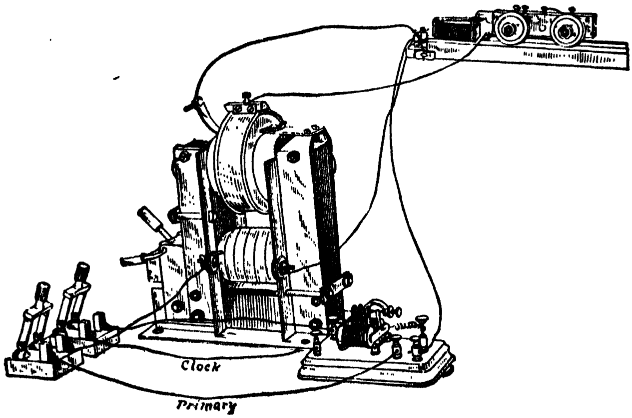


FIG. 8.

from $s = \frac{1}{2}at^2$ compute the value of the acceleration for each total distance s traveled, and average.

From the readings obtained compute the acceleration as follows:

Since the difference between the distances traveled by the body in two consecutive equal intervals gives the acceleration per that interval per that interval, obtain from the readings on the tape the average value of the acceleration. It is advisable to omit the first reading.

How to combine the reading so as to obtain the best result depends upon the number of readings; *e.g.*, in the case of nine readings the sum of the first three and the last three minus two times the sum of the middle three readings all divided by 3 gives the best average for the acceleration per thrice the interval per

thrice the interval. From this the acceleration per second per second may be readily obtained.

EXPERIMENT 7

EQUILIBRIUM OF PARALLEL FORCES

Apparatus.—A graduated bar (conveniently a meter stick) is suspended by two spiral springs whose supports are adjustable by means of turnbuckles; level; weight trays; and a number of known masses each of the order of a kilogram.

Method and Manipulation.—Determine the mass of the bar, and consider this mass as concentrated at the center of the bar.

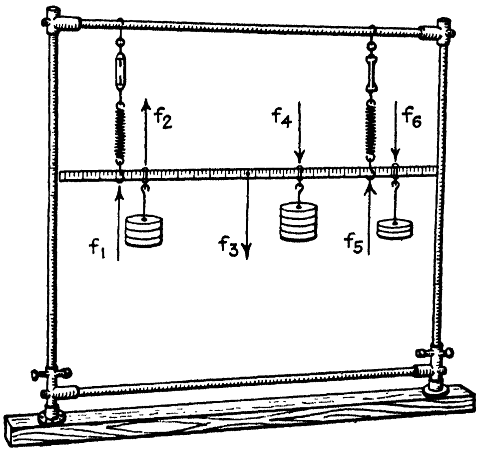


Fig. 9.

Suspend and measure the length of the two springs while they are hanging empty. This is done by placing the points of a divider in small holes, one at the upper end and one at the lower end of the spiral. The distance between the points is obtained from a steel scale. Suspend the bar by means of the two springs and stirrups placed a short distance from the ends of the bar. Suspend a tray carrying not less than 1 kg at each of three points between or outside the points of attachment of the springs.

By means of the turnbuckles adjust until the bar is level. Determine again the length of each spring; also read on the bar the point of attachment of the springs and trays.

Calibration of Springs.—In order to determine the force pulling on each of the springs it is necessary to determine the relation between the elongation of the spring and the force applied. To do this suspend the spring, and after measuring its length empty apply successively masses from 1 to 4 kg, and measure the length of the extended spring in each case. According to Hooke's law, the elongation of the spring should be proportional to the forces applied. Having obtained the proper data, plot a curve for each spring, using the masses as abscissas and the corresponding elongations as ordinates.

These calibration curves will now be used to determine the forces acting when the springs were attached to the bar.

First Static Condition of Equilibrium $\Sigma\ (f \cos \theta) = 0$.—Add algebraically all the vertical forces, expressed in grams weight, acting upon the bar, those acting upward being taken as positive. Determine the component of each of the forces in any other given direction, and obtain the algebraic sum.

The foregoing sums by the first static condition of equilibrium should equal zero. Any remainder obtained is a measure of the experimental error.

Second Static Condition of Equilibrium $\Sigma\ (fr \cos \theta) = 0$.—From the readings for the points of application of the forces obtain the distance r of each force from any arbitrary point on the bar. Obtain the algebraic sum of all the torques, calling the counter-clockwise torques positive.

By the second static condition of equilibrium this sum should equal zero. Note that this is true for any point on or off the bar.

EXPERIMENT 8

EQUILIBRIUM OF CONCURRENT FORCES

To test the condition for the equilibrium of forces that act at a common point.

Apparatus.—A force table with four pulleys adjustable about a horizontal graduated circle upon which their angular positions may be read; a connecting ring with centering pin; cords; weight pans; known masses.

Method and Manipulation.—Level the force table by placing the level parallel to a line through two of the feet. Then place

the level at right angles to the line mentioned above, and level by turning the third leveling screw.

Connect four unequal masses of not less than 200-g by means of strings to the center ring, as shown in Fig. 10. Adjust the angles until the ring will return to the center if tapped vertically after a small displacement. Record the masses and the corresponding angular readings. The angular readings must be taken counterclockwise from the zero of the circular scale.

Calculation.—Consider one of the four forces as the unknown. Assume the axis of X to pass through the zero of the scale. Obtain the sum of the X components, *i.e.*, Σx, of the three known

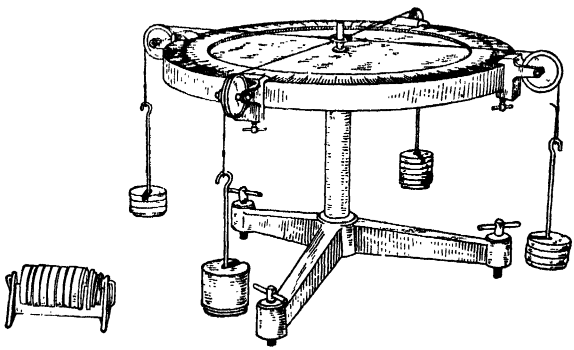


Fig. 10.

forces by multiplying each by the cosine of its counterclockwise angle and adding. Similarly, obtain the sum of the Y components Σy by multiplying each by the sine of its angle. The resultant is the square root of the sum of the squares of the preceding sums. The direction of the resultant is given by finding the angle that has a tangent equal to the ratio of the sum along Y to the sum along X.

The magnitude of the resultant and its direction as found above should check with the magnitude and direction of the unknown force.

Solve for the resultant graphically by laying off consecutively, according to an assumed scale, the three known forces, each in its proper direction as determined by means of a protractor. The magnitude of the line closing the polygon gives the magnitude of the resultant, and the direction of this line the direction of the

resultant. These again should check with magnitude and direction of the unknown force.

EXPERIMENT 9

EQUILIBRIUM OF NONCONCURRENT FORCES. LADDER

Purpose.—To test the conditions for the equilibrium of forces that do not act in directions passing through the same point.

Apparatus.—A model ladder supported in a vertical plane by horizontal wires at each end and by a vertical wire at the lower end; a weight to be hung on any rung of the ladder; spring dyna-

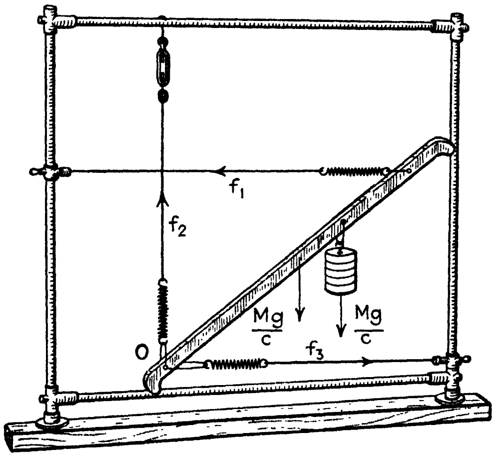


FIG. 11.

mometers by which the force along each wire may be measured; a large pair of calipers, or beam compasses, for obtaining the rectangular coordinates of the points at which forces act; a pair of dividers.

Method and Manipulation.—The ladder, with the weight hung upon it, is placed at any convenient angle with the horizontal, and the points at which the supporting wires are attached to the frame are adjusted until these wires are all either vertical or horizontal, *i.e.*, parallel to the frame. The coordinates of the points at which forces are applied to the ladder (including coordinates of the center of mass of the unloaded ladder, which is marked by a brass-headed nail) are measured by means of the caliper,

the left-hand and lower bars of the frame forming convenient axes of reference. The length of each spring dynamometer is measured three times by means of the divider and scale, as follows:

S_1 = the length when stretched by the force f_1 applied to the ladder,

S_2 = the length when stretched by a known force f (a set of weights hung on a weight pan),

S = the length when unstretched.

Then, assuming that the extension is proportional to the force producing it (Hooke's law), we have the following equation for f_1:

$$f_1 = f\frac{S_1 - S}{S_2 - S}.$$

An alternative method is to calibrate each spring as directed in Exp. 7 and determine the force f from the calibration curve. The weight of the load hung upon the ladder, as well as the weight of the ladder itself, must be obtained.

Calculations.—Plot on coordinate paper the positions of the points of application of all the forces acting on the ladder. Compute the amounts of the forces in the three wires.

First Static Condition of Equilibrium.—Find the sum, with proper regard to sign, of all the horizontal and of all the vertical forces.

Second Static Condition of Equilibrium.—Compute the total torque of each force about any arbitrary line, perpendicular to the plane of the forces, taken as the axis, *e.g.*, a line through 0 (Fig. 11), and obtain the sum of these torques.

All these results would be zero if the measurements were obtained with perfect accuracy.

EXPERIMENT 10

COEFFICIENT OF SLIDING FRICTION

Apparatus.—An inclined plane which may be clamped at any angle with the horizontal and provided with an adjustable pulley at the upper end; a heavy block of known mass, with cord, weight pan, and known weights for applying forces to the block in a

direction parallel to the plane; a steel scale for measuring the sides of the triangle formed by three points marked on the edges of the inclined plane and its horizontal base; several flat sheets of different materials.

Method and Manipulation.—Place the surface under investigation upon the inclined plane, and place the block upon it. Connect a known mass to the cord passing over the pulley, and adjust the inclination of the plane until the block will just continue to

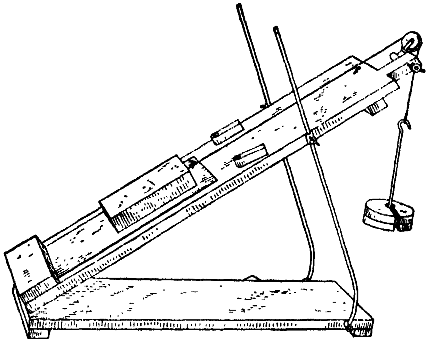


Fig. 12.

slide up the plane (without acceleration) when given a slight initial motion in that direction. Measure the sides of the triangle formed by the marked points on the edges of the inclined plane and its base. Readjust, using the same known mass if possible, until the block will just continue to slide down the plane (without acceleration) when given a slight initial motion in that direction. Measure the sides of the new triangle now formed by the marked points. Care should be taken to elevate or depress both sides of the plane by the same amount in making an adjustment and that the cord passing over the pulley is parallel to the plane. Repeat with each of the different pairs of surfaces available.

Calculations.—Construct a triangle with the three measured lengths found in each case tested, using as large a scale as possible and placing the side corresponding to the base of the plane in a horizontal position on the paper. Construct the force polygon for each case as follows:

1. Through any point on the side of the preceding triangle which corresponds to the inclined plane draw a vertical line, and

lay off upon it, to a convenient scale, the weight of the block (which will be given).

2. From line 1 lay off up the inclined side of the triangle the weight hung over the pulley, using the same scale as in 1.

3. From the upper end of line 1 draw a line parallel to the inclined side.

4. From the upper end of line 2 draw a perpendicular to the inclined side intersecting line 3.

The polygon will now be complete; indicate the directions of the four forces by arrowheads. Consider the segment on the line in construction one above as side 1 of this polygon; that in construction 2, as side two; etc. Side 4, the fourth consecutive side of the polygon, represents the force of friction at the pulley and must be subtracted from side 2. The coefficient of sliding friction is therefore the ratio of the difference between the second and the fourth sides of the polygon divided by the third side.

Find the coefficient of sliding friction between various pairs of surfaces.

EXPERIMENT 11

LOCATION OF THE CENTER OF MASS OF A WHEEL

Apparatus.—A wheel and axle; a leveling way or track along which the axle may roll freely; a double plumb line; weight of the order of 5-g.

Method and Manipulation.—1. Level the track upon which the wheel rests.

2. Hang the double plumb line over the shaft; let the wheel vibrate until it comes to rest; and carefully mark the points aa' at which the plumb lines cross the rim of the wheel.

3. Hang a known mass m from the rim of the wheel by means of a string passing over the circumference; a small amount of soft wax will serve for attaching the string. Again let the wheel vibrate and come to rest, and mark the points bb' at which the plumb lines cross the rim. Measure the arcs ab and $a'b'$.

4. Repeat 3 with the mass m suspended on the opposite side of the wheel so as to eliminate to some extent errors due to inaccurate leveling of the track.

5. Determine M, the mass of the wheel.

6. Measure the radius R of the wheel.

7. With a flexible steel tape measure the arcs ab and $a'b'$, and average these with the corresponding arcs obtained from the reverse suspension.

Computations and Results.—Let A be the point midway between the two points aa', and B the point midway between the points bb'. A line drawn from O, the center of the wheel, to A passes through the center of mass of the wheel, and the angle

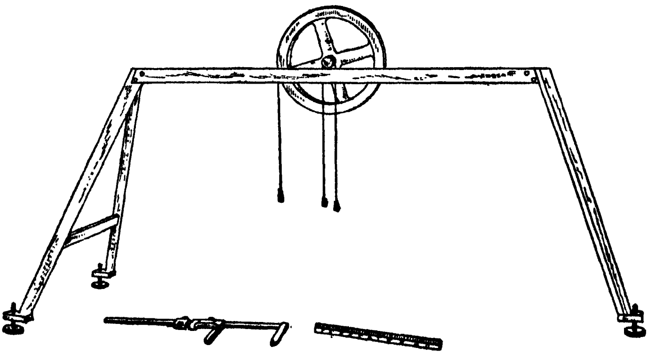


FIG. 13.

AOB is a measure of the angle through which the center of mass of the wheel has been displaced. The distance x of the center of mass from the axis O is determined by the torque equation

$$mgR = Mgx \sin \overline{AOB},$$

in which m, M, and R are known, and the angle $\overline{AOB}$ is determined in degrees from the average of the arcs ab and $a'b'$ multiplied by $360/2\pi R$. Compute x, and from its value determine the mass that must be removed from the rim of the wheel in order to cause it to balance.

EXPERIMENT 12

TO TEST HOOK'S LAW AND TO DETERMINE YOUNG'S MODULUS IN THE CASE OF THE BENDING OF A ROD

Apparatus.—A bar supported by a fulcrum at each end and provided with a weight pan at the center; a stage micrometer and microscope arranged for measuring vertical distances; a number of weights.

Method and Manipulation.—Adjust the rod so that it lies firmly on the fulcrums. Focus the microscope, and turn the micrometer until the cross hair is on the fine reference point on the rod, being careful to approach the point so as to avoid the error due to backlash of the micrometer screw. Obtain the micrometer reading R_0, the weight pan being empty. Add a known mass to the pan; adjust; and again read the micrometer. The difference between R_0 and the reading after a load has been added gives the deflection for that load. Continue thus until six or eight known masses have been added. Repeat the preceding observations by again beginning with no load in the weight pan. The average of the deflections obtained in three runs should give a satisfactory

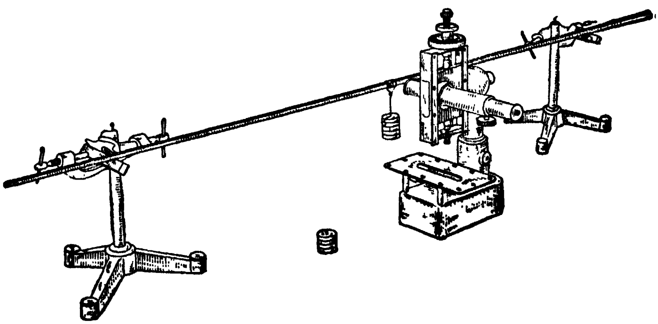


Fig. 14.

result. Care must be exercised so as not to jar the apparatus in the meantime.

Graphical Interpretation of Observations.—The relation between the loads and deflections may best be shown by the use of coordinate paper. Plot the loads as abscissas and the one-half, first, and the second powers of the deflections as ordinates. Also, plot the deflections against the loads on double log paper and evaluate the intercept. (See Graphical Representation of Results in introductory chapter.)

If the curve is a straight line in the case of the first power, or if the log curve is a straight line at an angle of 45 deg, the relation is that the deflections vary directly as the loads. This proportionality is expressed by Hooke's law, *i.e.*, the stress is proportional to the strain.

Young's Modulus.—In the case of a rectangular bar of depth d, breadth b, and length S, supported at both ends, Young's modulus

$$Y = \frac{fS^3}{4bd^3y},$$

where f is the force in dynes applied at the middle and y is the deflection at the middle. Measure the dimensions of the bar, and compute Y for each of three values of f and y determined from the graph.

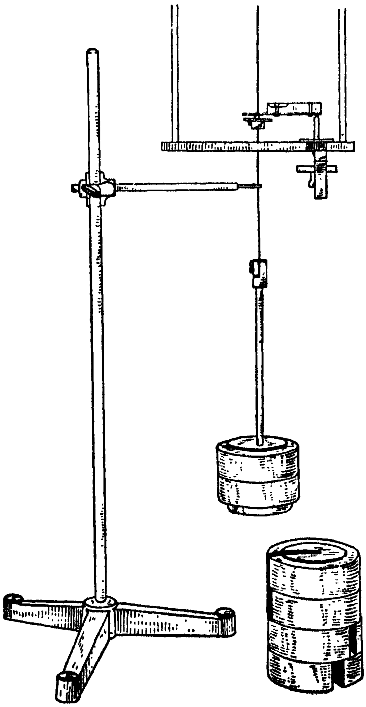


FIG. 15.

EXPERIMENT 13

TO TEST HOOKE'S LAW AND TO DETERMINE YOUNG'S MODULUS IN THE CASE OF THE STRETCHI'G OF AN IRON WIRE

Apparatus.—The wire is attached to the ceiling and hangs vertically. It has a pan for weights attached at the lower end. A cross bar, clamped to the wire near the upper end, supports two vertical rods which, in turn, support a cross bar near the lower end of the wire through which the wire passes freely. This bar supports, near one end, a vertical screw provided with a micrometer head, the head being beneath the bar. Above the bar a circular disk is clamped to the wire. A metal plate, provided with a level on top and two pointed feet beneath one end, is placed so that the feet rest on the circular disk and the other end rests on the upper end of the micrometer screw.

Method and Manipulation.—1. *To Test Hooke's Law, i.e., That Stress Is Proportional to the Strain.*—Place a mass of 1 kg in

the weight pan, and adjust the micrometer until the level reads zero. Read the micrometer. This gives the zero reading R_0, the wire being straight. Add an additional kilogram and again level and obtain the micrometer reading R. $R - R_0$ gives the elongation, or strain, for the stress represented by the second weight. Continue in this manner until deviation from Hooke's law begins, *i.e.*, the elastic limit has been reached. Prepare a graph of the foregoing results by laying off the elongations as ordinates and the corresponding loads as abscissas. The straight portion of the graph represents the region over which Hooke's law holds.

2. *To Determine Young's Modulus and the Elastic Limit.*—Young's modulus is the stress per unit cross-sectional area divided by the strain per unit length of the wire, *i.e.*,

$$Y = \frac{fs}{Ae},$$

where f represents the total load in dynes or poundals, e the total elongation produced by the force f, s the length, and A the cross-sectional area of the wire measured in terms of the centimeter or the inch.

Evaluate Young's modulus for a given load by obtaining the elongation from the graph.

Evaluate the load per unit area at the elastic limit.

EXPERIMENT 14

ELONGATION OF A SPIRAL SPRING

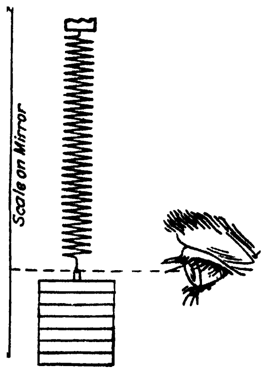


Fig. 16.

Apparatus.—A spiral spring, carrying a pan for weights, is suspended in front of a mirrored scale.

Method.—Readings are taken on the mirrored scale opposite the point of the lower hook when no weights are in the pan and after each additional weight until about four have been added. In order to avoid error due to parallax each reading must be taken when the eye is in line with the point of the hook and its image in the mirror.

Plot the readings obtained, and determine if Hooke's law holds within the limits of observational error.

EXPERIMENT 15

HOOKE'S LAW AND SIMPLE HARMONIC MOTION

Apparatus.—Same as in Exp. 14; and a stop watch.

Method and Manipulation.—In the preceding experiment it was found that

$$\frac{\text{Stress}}{\text{Strain}} = K, \text{ a constant.}$$

When a vertical spring carrying a constant load is elongated beyond the point of rest, the upward contracting force is greater than the force of gravity acting downward upon the attached mass. If the spring is shortened, the force of gravity is greater than the contracting force. At the point of rest the two forces are equal.

For any displacement x of the spring from the point of rest, there is a force tending to restore it which is equal to ma, where a is the acceleration that it gives to the attached mass m.

From Hooke's law and as shown in the last experiment,

$$-\frac{ma}{x} = K_1 \qquad \text{or} \qquad a = -K_1 x.$$

The negative sign is introduced because ma and x are oppositely directed. The foregoing is the condition for simple harmonic motion for which the general equation for the period T is

$$T = 2\pi\sqrt{-\frac{x}{a}}.$$

If the mass m produces a static elongation s, then when the mass m is vibrating vertically and is at an upward displacement equal to s, the acceleration downward is g; therefore, at this instant

$$-\frac{x}{a} = \frac{s}{g}.$$

The equation for the period thus becomes

$$T = 2\pi\sqrt{\frac{s}{g}}.$$

Determine from the curve obtained in Exp. 14 the elongation s which a mass m will produce.

Substitute the value of s in the equation, and compute the period T. Using the mass m which produced the elongation s, cause it to vibrate vertically through a small amplitude, and determine the time of vibration or period by counting the number of vibrations in a time interval determined by means of the stop watch. These two values for the period T should agree within the limits of experimental error.

EXPERIMENT 16

TORSION OF A ROD

Apparatus.—A rod is firmly clamped at one end. At the other end of the rod there is a graduated wheel on which the angle of

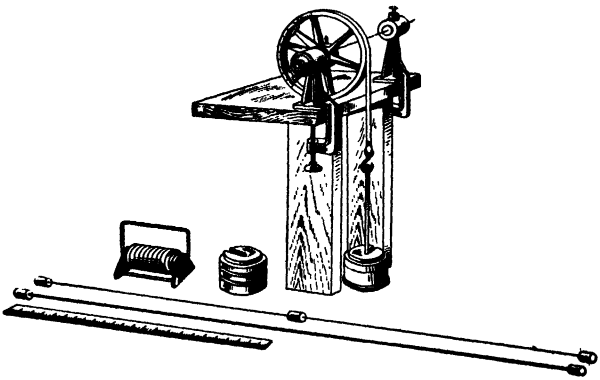


Fig. 17.

torsion may be read. Attached to the circumference of the wheel is a ribbon to the free end of which a weight pan is connected.

Method and Manipulation.—Take the reading on the wheel when the pan is empty and after each additional load until several have been added. Care must be taken to avoid parallax and friction between the various parts.

Plot the readings obtained, and determine whether or not Hooke's law holds.

EXPERIMENT 17

HOOKE'S LAW AND SIMPLE HARMONIC MOTION

Apparatus.—Same as in Exp. 16; and in addition a stop watch and a metal disk.

Method.—Since, as found in the preceding experiment, the strain is proportional to stress, the rod, when set in motion as a torsional pendulum, will vibrate with simple harmonic motion. In order to compute the time of vibration, the general equation for the period of a simple harmonic motion

$$T = 2\pi\sqrt{-\frac{x}{a}}$$

must be altered so as to involve quantities that can in this case be readily measured.

For an angular displacement θ, a point at a distance r from the axis has a linear displacement $x = r\theta$. Likewise, if α is the angular acceleration, the linear acceleration at a distance r is $a = r\alpha$,

$$\therefore \frac{x}{a} = \frac{r\theta}{r\alpha} = \frac{\theta}{\alpha};$$

$$\therefore T = 2\pi\sqrt{-\frac{\theta}{\alpha}}.$$

The moment, or torque, of the couple necessary to produce a displacement θ is $K\theta$, where K is the moment of torsion, *i.e.*, the moment, or torque, of the couple necessary to produce an angular displacement of one radian; but

$$-K\theta = \alpha I,$$

where I is the moment of inertia of the system.

$$\therefore \frac{x}{a} = \frac{\theta}{\alpha} = -\frac{I}{K},$$

and

$$T = 2\pi\sqrt{\frac{I}{K}}.$$

If, therefore, I and K are known, the period T can be determined.

To obtain K determine from the curve in Exp. 16 the number n of circumference divisions in the angle of torsion produced by a mass m. This angle θ, in radians, equals $\frac{n}{N}2\pi$, where N is the number of divisions in the whole circumference. If r is the radius of the graduated disk, then

$$K\theta = mgr,$$
$$\therefore K = \frac{mgr}{\theta}.$$

Compute I from the dimensions of the cylinder.

Substitute the values of K and I in the equation, and solve for T.

Remove the rod, attach disk, and suspend from bracket so as to form a torsion pendulum. Determine the time of vibration. This should check with the computed time if the vibration is simple harmonic.

From the equation

$$\mu = \frac{128\pi SI}{T^2 d^4}$$

compute the coefficient of simple rigidity of the material of the wire.

In the foregoing equation S is the length of the wire, and d its diameter.

EXPERIMENT 18

MOMENT OF INERTIA OF AN IRREGULAR BODY ABOUT AN AXIS THROUGH ITS CENTER OF MASS

Method.—When the body is suspended by a wire whose moment of torsion is K, its time of vibration as a torsional pendulum is

$$T = 2\pi\sqrt{\frac{I}{K}}. \qquad \text{(See Exp. 17.)}$$

If a body of known moment of inertia I' is suspended from the same wire, the time of vibration

$$T' = 2\pi\sqrt{\frac{I'}{K}};$$
$$\therefore I = \frac{T^2}{T'^2}I'.$$

Fig. 18.

Apparatus.—The body whose moment of inertia is to be determined; a body whose moment can be calculated; a steel wire with proper attachments.

Manipulation.—Determine the time of vibration for each of the bodies to an accuracy comparable with the accuracy in the other measurements. Compute I' from the mass and dimensions of the body. Solve for I.

EXPERIMENT 19

TO TEST EXPERIMENTALLY THE EQUATION $I = I_0 + Mh^2$

Method.—The moment of inertia I of a body may be determined experimentally by adding it to a torsional pendulum of known moment of inertia I' and known time of vibration T' and obtaining the new time of vibration T.

$$\text{Thus } T' = 2\pi\sqrt{\frac{I'}{K}}, \qquad T = 2\pi\sqrt{\frac{I + I'}{K}};$$

$$\therefore I = \frac{T^2 - T'^2}{T'^2} I'.$$

If two similar bodies of such form that their moments of inertia can be computed from their masses and dimensions are placed symmetrically on opposite sides of the torsional pendulum so as not to disturb its position, the equation $I = I_0 + Mh^2$ can be tested experimentally.

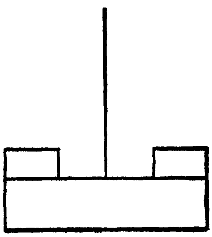


FIG. 19.

Apparatus.—The torsion pendulum used in Exp. 17; two small cylinders.

Manipulation.—Obtain by computation from its dimensions the moment of inertia I' of the torsional pendulum, and determine its time of vibration.

Place the two cylinders on opposite sides of the pendulum so that their centers are equidistant from and in line with the wire, and obtain the time of vibration with the same degree of accuracy as above.

By means of the values I', T, and T' thus obtained, compute I, the moment of inertia of the cylinders about the axis around which they rotated.

Obtain the mass and dimensions of the cylinders and their distance h from the axis of rotation. The value of I obtained by substituting these values in the equation $I = I_0 + Mh^2$ should agree to within the limits of experimental error with the experimental result.

EXPERIMENT 20

ANGULAR ACCELERATION

Apparatus.—A heavy disk is mounted so that it may be rotated about its axis. A tuning fork is placed so that its vibrations may be traced by means of a stylus on one of the faces which has been coated with a solution of calcium carbonate. The other face of the disk has a circle which is graduated in degrees. On each side of the disk is a magnifying glass fitted with a cross hair. By the aid of these, the length of a given number of wave lengths on the tracing may be determined in degrees on the graduated circle.

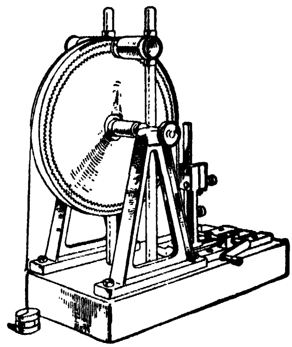


Fig. 20.

Method and Manipulation.—By means of a small amount of wax attach a string, and wind it on the circumference of the disk. To the free end of the string attach a mass M of about 100-g. Adjust the disk so that the stylus will trace the vibrations of the fork on the coated surface. Let the weight start the cylinder from rest, and then move the fork slowly so that the tracings will not overlap. As the speed of the disk increases, the traced vibrations become longer. Let θ, θ', θ'', etc., be the readings on the graduated circle, counting from zero, obtained at the beginning and end of consecutive lengths of about 40 vibrations. Let n be the frequency of the fork. The difference between two consecutive readings gives the angular distance in degrees that the disk traveled during an interval of 40 vibrations, or $40/n$ sec.

The difference k between two of these consecutive distances is the angular acceleration in degrees per $40/n$ sec. per $40/n$ sec.

Obtain an average value for k from several measurements by combining the readings so as to avoid cancellation (see Introduction and Exp. 6). For example, in the case of eight readings, one-fourth of the difference between the sum of the first and last two and the sum of the others gives the best average. From k compute α, the angular acceleration in radians per second per second.

EXPERIMENT 21

TO TEST EXPERIMENTALLY THE RELATION $Fr = \alpha I$

Apparatus.—Same as in Exp. 20.

Method and Manipulation.—Obtain α, the positive angular acceleration of the disk, in the manner explained in Exp. 20.

In like manner obtain α_1, the negative acceleration, *i.e.*, the acceleration after the weight was released and the speed of the disk was decreasing owing to friction alone.

Since the mass M descends with an acceleration a, the torque acting on the cylinder is

$$L = M(g - a)r,$$

or, since $a = \alpha r$,

$$L = M(g - \alpha r)r,$$

where g is the acceleration due to gravity and r is the radius of the disk on which the string supporting the mass M is wound.

Of this torque an amount equal to $I\alpha_1$ is required to overcome the frictional resistance, and the remainder is effective in producing motion. Therefore the equation to be tested is

$$M(g - \alpha r)r = I(\alpha + \alpha_1).$$

The value of the moment of inertia I is computed from the expression for the moment of inertia of a cylinder about its axis.

I multiplied by $(\alpha + \alpha_1)$ gives the value of the second member which should check with the first.

EXPERIMENT 22

ANGULAR VELOCITY

Apparatus.—Same as in Exp. 20.

Method and Manipulation.—Wind the string about the disk, and note the height of the mass M. Release the disk, and the instant that M strikes the floor let the stylus trace the vibrations of the fork on the smoked surface. Repeat several times. The length of one of the traced vibrations is the angular distance in degrees that a particle on the disk traveled during one period of the fork,

If θ = the length of one vibration in deg,
n = the frequency of the fork,
ω_1 = the angular velocity in deg/sec,
then $\omega_1 = n\theta$.

In obtaining a value for θ, measure the length of several vibrations, and divide by the number.

Since on account of friction the velocity of the disk is not uniform, care must be taken to measure the vibrations, in each case, corresponding to the instant that the weight strikes the floor.

From ω_1 determine ω, the angular velocity in radians per second.

EXPERIMENT 23

TO SHOW THAT $Fs = \frac{1}{2}\omega^2 I$

Apparatus.—Same as in Exp. 20.

Method and Manipulation.—The object is to show that the force F producing rotation, from rest, in a body multiplied by the distance s through which it acts is equal to the kinetic energy possessed by the body when the force ceases to act, *i.e.*,

$$Fs = \frac{1}{2}\omega^2 I,$$

where ω = the angular velocity,
I = the moment of inertia of the body.

F, the effective force, must be found by taking friction into account, *i.e.*, by use of

$$M(g - a)r - I\alpha' = Fr \qquad \text{(Exp. 21)}$$

Obtain ω (Exp. 22) and measure s. Compute I from the mass and dimensions of the disk. Substitute in the equation, and determine whether the equality holds within the limits of experimental error.

EXPERIMENT 24

TO PROVE THAT THE MOMENTUM IS THE SAME AFTER AS IT IS BEFORE AN INELASTIC IMPACT

Apparatus.—Two metal cylinders are suspended from the ceiling by parallel wires, as shown in Fig. 21. One of the cylinders is provided with fine points which during collision penetrate the lead surface of the other, thereby preventing the two from separating and giving the equivalent of inelastic collision.

Method and Manipulation.—Adjust the cylinders until they are horizontal and in line with each other. Place the table so that when the cylinders are swinging, the vertical rod beneath B moves parallel and close to the wire cd. The wire cd is leveled by

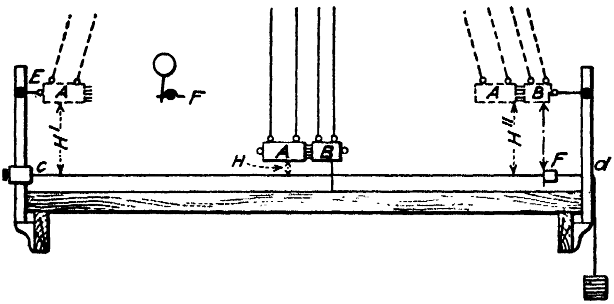


FIG. 21.

adjusting so that it is even with the surfaces of the mercury contained in two communicating glass tubes.

By means of a vertical mirrored scale obtain the height H. Raise cylinder A, and attach it at E. Obtain the height H'. Release A. After collision the two cylinders move on together, and the vertical rod beneath B will leave at a definite point the paper index f placed on the wire cd. By means of a thread draw the two bodies over until the vertical rod just touches the paper index, and obtain the heights H''.

The horizontal velocity of A just before impact is $v = \sqrt{2g(H' - H)}$, and the velocity of A and B after impact is $v' = \sqrt{2g(H'' - H)}$.

If the momentum is the same before and after impact, then

$$mv = (m + m')v',$$

or

$$m\sqrt{2g(H' - H)} = (m + m')\sqrt{2g(H'' - H)};$$
$$\therefore m\sqrt{H' - H} = (m + m')\sqrt{H'' - H}.$$

From the data obtained, compute each member of this equation, and compare.

The height of the two cylinders is best obtained by measuring the height of A and of B and averaging the two.

EXPERIMENT 25

TO DETERMINE THE CENTRIFUGAL FORCE ACTING UPON A ROTATING BODY

Apparatus.—A cylindrical body of mass M (about 150-g) is connected by means of a spiral spring to the opposite end of a horizontal frame which may be rotated about a vertical axis through its center. As the frame rotates, the centrifugal force acting on the cylindrical mass elongates the spiral spring until a lever index indicates a definite radius. A Veeder revolution counter is actuated by the rotating vertical shaft. A friction disk permits alteration in the speed of rotation.

Method and Manipulation.—Adjust the speed until the index indicates a definite radius for the cylindrical mass. Read the countèr at the beginning and end of a 5-minute interval as determined by a stop clock or a timing signal.

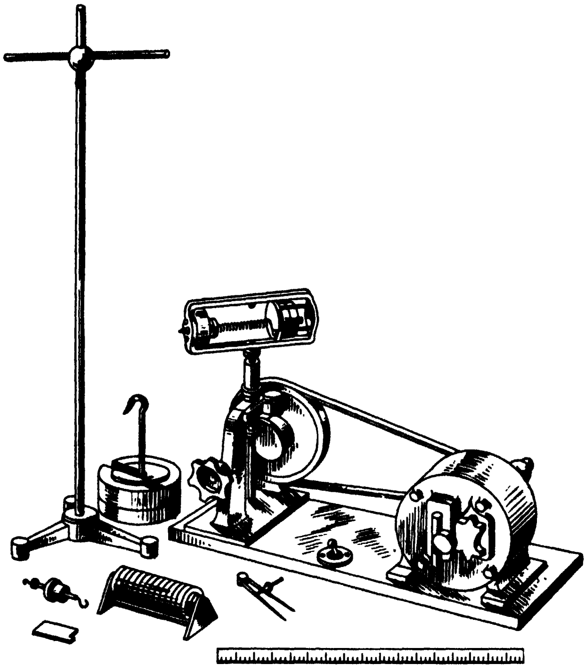
Repeat three times, and obtain N, the average number of revolutions per second. The angular velocity is then

$$w = 2\pi N \text{ radians/sec.}$$

By means of a threaded hook provided with a nut which may be turned by hand, move the cylinder away from the axis of rotation until the index indicates the same radius as when rotating. By means of a divider determine the distance R from the axis of rotation to the median plane of the cylinder as determined by a square plate whose line is placed even with the median plane. The centrifugal force f acting may now be evaluated from the expression

$$f = Mw^2R.$$

To verify the result suspend the spring frame and cylinder. Attach a pan for weights, and determine the total weight M_1g when the index indicates the same elongation of the spring as



before. The weight M_1g should check within the experimental error with f, as determined above.

EXPERIMENT 26

ACCELERATION DUE TO GRAVITY BY THE PHYSICAL PENDULUM METHOD

Apparatus.—A rectangular bar, which has, near one end, an axis perpendicular to its plane and is about 6 by 25 mm in cross section and about 60 cm long. This bar has a yoke, provided with two knife-edges and which may be clamped at any point on the bar, the bar being suspended in such a manner that the

ends of the knife-edges rest equally on the supporting surface at each side; a stop clock.

Method and Manipulation.—Clamp the supporting yoke at the upper end of the bar, and obtain the time of vibration with a probable error not greater than 0.1 per cent. This may be obtained by tallying on a Veeder counter the number of swings in an interval of about 5 minutes.

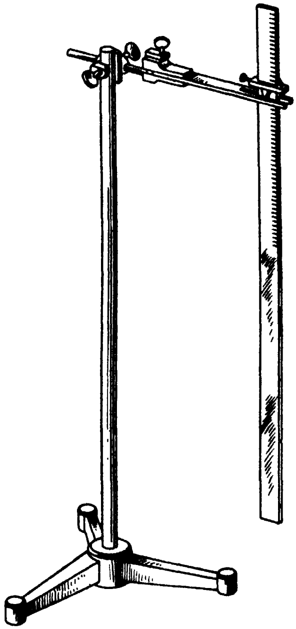


Fig. 23.

Measure the length and width of the bar.

If T = the period,

I = the moment of inertia of the pendulum about the axis of suspension,

h = the distance from the axis to the center of gravity of the pendulum,

M = the mass of the bar,

then

$$T = 2\pi\frac{\sqrt{I}}{\sqrt{Mgh}}$$

and

$$g = \frac{4\pi^2 I}{MhT^2}.$$

Using this expression and the data obtained as directed above, compute the value of g.

Determine accurately the distance from the line of the knife-edges to the upper end of the bar. For this purpose it is well to use the stage micrometer required in Exp. 12.

Then move the yoke so that the knife-edges are 2 cm farther from the upper end of the bar than in the former position. Again obtain the period as before. Continue in this manner until about 10 periods have been obtained. Then plot the periods as ordinates and the distances of the axis from the upper end of the bar as abscissas. A U-shaped curve showing a minimum period will be obtained. From a point on the Y axis somewhat removed from the minimum period draw a line parallel to the X axis.

This line will intersect the curve at two points of equal period. Determine the X coordinates for these two points, and subtract their sum from the length of the bar. This gives the length of the equivalent simple pendulum S. From S and the period T compute g, the acceleration due to gravity from the relation

$$T = 2\pi\sqrt{\frac{S}{g}}.$$

EXPERIMENT 27

TO DETERMINE THE INERTIAL MASS OF A BODY

Apparatus.—A horizontal circular disk is mounted so that it may be rotated about a vertical axis through its center. A speed of the disk of three revolutions per second is produced by means of a synchronous motor provided with a suitable reduction gear.

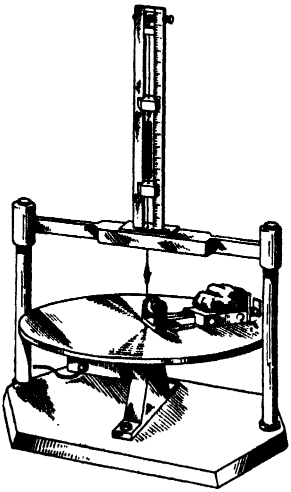


FIG. 24.

A car on roller bearings is mounted over a radial guide. From the car a string passes under a pulley, mounted on ball bearings, and axially up to a swivel which, in turn, is connected to the lower end of a vertical spiral spring. At the lower and upper ends the spring is connected to guides which have index lines for indicating the positions of the ends on the vertical scale at the side of the spring. The spring is supported by means of a string which passes over a friction roller at the top on which the string may be wound.

Method and Manipulation.—Place a standard body of known mass M_0 on the car, and measure the distance R_0 of its center of mass from the axis of rotation when the lower index is at some convenient value S_0.

Cause the disk to rotate, and adjust the spring so that the lower index is at S_0 on the scale, and determine the position S_1 of the upper index. The spring has now been stretched a distance

$S_1 - S$, where S is the position of the upper index when the spring is unstretched. S need not be determined. If K is the constant of the spring, *i.e.*, the force in gravitational units required to elongate the spring a unit distance, then

$$K(S_1 - S) = \frac{M_0 w^2 R_0}{C} + \frac{M_c w^2 R_c}{C}, \qquad (A)$$

where M_c is the mass of the car, R_c is the distance of the center of mass of the car from the axis of rotation, and w is the angular velocity.

Remove the known mass M_0; allow the disk again to rotate. Place the lower index at S_0 as before, and determine S_2, the position of the upper index on the scale. Then

$$K(S_2 - S) = \frac{M_c w^2 R_c}{C}. \qquad (B)$$

Subtract Eq. (B) from Eq. (A). This gives

$$K(S_1 - S_2) = \frac{M_0 w^2 R_0}{C};$$

$$\therefore K = \frac{M_0 w^2 R_0}{(S_1 - S_2)C}.$$

From this expression K may be numerically determined from the values obtained as directed above.

To Determine an Unknown Mass

Place the body, of mass M to be determined, on the car and against the outer end. Cause the disk to rotate. Set the lower index at S_0 as before, and read the position S_1' of the upper index on the scale. Move the body a distance d nearer the axis of rotation. Again cause the disk to rotate, and when the lower index has been adjusted to S_0 read the position S_2' of the upper index. From these two settings we have

$$K(S_1' - S) = \frac{M w^2 R_1}{C} + \frac{M_c w^2 R_c}{C},$$

and

$$K(S_2' - S) = \frac{M w^2 R_2}{C} + \frac{M_c w^2 R_c}{C}.$$

Subtracting gives

$$K(S_1' - S_2') = \frac{Mw^2}{C}(R_1 - R_2);$$

$$\therefore\ M = \frac{CK(S_1' - S_2')}{w^2(R_1 - R_2)}.$$

Since $R_1 - R_2 = d$ from the setting, the expression for the unknown mass is

$$M = \frac{CK}{w^2}\frac{(S_1' - S_2')}{d},$$

or

$$M = K_0\frac{(S_1' - S_2')}{d},$$

where K_0 stands for CK/w^2.

A similar procedure in the case of a known mass M_0 gives

$$M_0 = K_0\frac{(S_1'' - S_2'')}{d}.$$

Therefore

$$\frac{M}{M_0} = \frac{(S_1' - S_2')}{S_1'' - S_2''}.$$

This procedure avoids the necessity of determining K.

If $\dfrac{M_0}{(S_1'' - S_2'')}$ is represented by k, a constant, which may be determined once and for all, the mass desired becomes

$$= k(S_1 - S_2).$$

Determine the mass M from the observed values, and compare with the gravitational mass as determined by means of a beam balance.

CHAPTER II

FLUIDS

EXPERIMENT 1

SPECIFIC GRAVITY OF SOLIDS

Apparatus.—Specific gravity bottle; chemical balance; distilled water; the solid whose specific gravity is to be determined.

Method.—Obtain the mass m of the substance. Use the greatest amount of the solid that can be placed in the bottle without interfering with its stopper. Fill the bottle with distilled water to a fixed point in the capillary tube. Wipe dry, and obtain its mass M. Place the substance in the bottle, and have the water extend to the same point in the capillary tube, and obtain the mass M'. Air bubbles adhering to the solid must be removed.

The specific gravity

$$\sigma = \frac{m}{m + M - M'}.$$

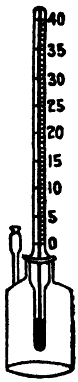


FIG. 1.

Precautions.—The tube leading from the bottle must be closed to prevent evaporation. As the volume of the bottle and the density of the water change with temperature, care must be exercised to have the temperature of the water the same at each filling.

EXPERIMENT 2

SPECIFIC GRAVITY OF SOLIDS BY IMMERSION

Apparatus.—An analytical balance; distilled water; the solid whose specific gravity is to be determined; a glass beaker and a stand for it which may be placed so that the beaker is over the left-hand scale pan; a fine wire for suspending the solid.

Method and Manipulation.—Determine the mass M of the solid whose specific gravity is desired. Suspend the solid from

the hook above the scale pan by means of the fine wire, and determine the mass M_1 of the wire and solid in air. Place the beaker containing the distilled water on the stand, and adjust so that the solid is immersed. Great care must be taken to remove any air bubbles adhering to the solid. Determine the apparent mass M_2 of the wire and solid. The specific gravity is

$$\sigma = \frac{M}{M_1 - M_2}.$$

For more accurate determination the temperature of the water must be taken into consideration.

EXPERIMENT 3

TO DETERMINE THE SPECIFIC GRAVITY OF A LIQUID BY MEANS OF A MOHR'S BALANCE

Apparatus.—A Mohr's balance consists of a beam supported on steel knife-edges. One half of the beam is graduated to tenths. Attached to the other half is a counterweight so that the beam will balance in air when a sinker S is suspended at the end of the tenth division of the graduated arm. If the balance is imperfect, it should be adjusted by the addition of a small weight (piece of paper) to the proper side before any readings are attempted. A rider is also provided which, when attached to the beam at the end of the tenth division, will cause the beam to balance when the sinker is immersed in water at 15°C. The volume of the sinker is 5 cc, so the mass of the largest rider is 5-g. By Archimedes' principle, this rider is then equal in mass to the mass of the water displaced. Four other riders are also provided which have masses equal to 1.0, 0.1, 0.01, 0.001 of the mass of the rider mentioned. If the temperature corrections are not to be made, the 0.001 rider will not be supplied. The sinker is provided with a thermometer so that the temperature at which it is used may be determined.

Method and Manipulation.—The specific gravity in this case is defined as the ratio of the density of the liquid to the density of water at 15°C. Suspend the sinker in the liquid the specific gravity of which is desired, and add riders until a balance is obtained. Then, if the riders are distributed as shown in Fig. 2,

where a is the unit rider, b the 0.1, and c the 0.001, the specific gravity is 1.2608 if the temperature of the liquid is 15°C.

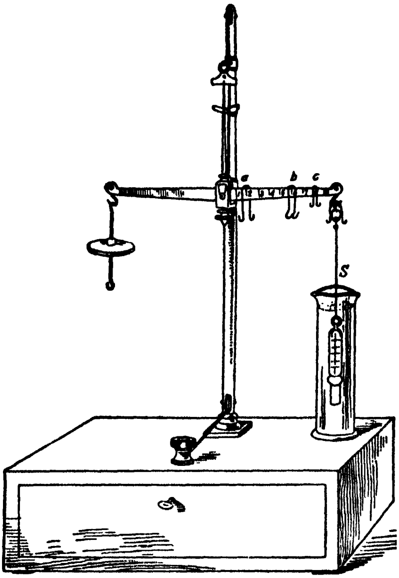


Fig. 2.

If, however, the temperature of the liquid is $t°$, and the reading of the balance is m_t, the mass of the liquid displaced by the sinker is

$$K \cdot m_t = V_0(1 + \alpha t)\rho_t, \tag{1}$$

where V_0 = volume of sinker at 0°C.,
α = the coefficient of expansion of sinker,
K = the mass of the unit rider,
ρ_t = the density of the liquid at temperature t.
Likewise for water at 15°C.

$$Km_{w15} = V_0(1 + \alpha 15)\rho_{w15}, \tag{2}$$

where m_{w15} = reading of balance with water at 15°C.,
ρ_{w15} = density of water at 15°C.

Dividing Eq. (1) by Eq. (2) gives

$$\frac{m_t}{m_{w15}} = \frac{(1 + \alpha t)\rho_t}{(1 + \alpha 15)\rho_{w15}}. \tag{3}$$

Defining specific gravity with reference to water at 15°C. gives

$$\sigma = \frac{\rho_t}{\rho_{s15}} = \frac{m_t(1 + \alpha 15)}{m_{s15}(1 + \alpha t)},$$

or, since $m_{s15} = 1$ by construction of balance,

$$\sigma = m_t \frac{(1 + \alpha 15)}{(1 + \alpha t)}.$$

Defining specific gravity with reference to water at 4°C. gives

$$\sigma = \frac{\rho_t}{\rho_{s4}},$$

where ρ_{s4} = density of water at 4°C.

Solving Eq. (3) for ρ_t gives

$$\rho_t = \frac{m_t(1 + \alpha 15)\rho_{s15}}{m_{s15}(1 + \alpha t)};$$

$$\therefore \sigma = \frac{m_t(1 + \alpha 15)}{m_{s15}(1 + \alpha t)} \frac{\rho_{s15}}{\rho_{s4}},$$

or

$$\sigma = m_t \frac{(1 + \alpha 15)}{(1 + \alpha t)} \frac{\rho_{s15}}{\rho_{s4}}, \qquad \text{since } m_{s15} = 1.$$

Taking $\rho_{s15}/\rho_{s4} = 1$ gives, as before,

$$\sigma = m_t \frac{(1 + \alpha 15)}{(1 + \alpha t)}.$$

Since α is small, $\dfrac{1 + \alpha 15}{1 + \alpha t}$ is nearly = 1. With these approximations we then have

$$\sigma = m_t.$$

The distribution of the riders on the graduated arm which causes the beam to balance when the sinker is immersed gives therefore the approximate value of the specific gravity of the liquid at any temperature. It is to be noted that the specific gravity of the liquid with reference to water at the same temperature is

$$\sigma = \frac{m_t}{m_{s}t},$$

which is the ratio of the readings of the balance when the sinker is in the liquid or in water both at the same temperature.

Determine the approximate and the exact specific gravities of the solution provided for the experiment.

EXPERIMENT 4

TO CALIBRATE A HYDROMETER SCALE OF EQUAL DIVISIONS

Apparatus.—A hydrometer is an instrument for determining directly the density or specific gravity of a liquid. It consists of a glass tube which is weighted with mercury or lead at the lower end and which has a thinner portion or stem at the other end which is graduated in equal divisions or in unequal divisions which give directly the density or specific gravity. Both graduations will be found on the hydrometer to be used.

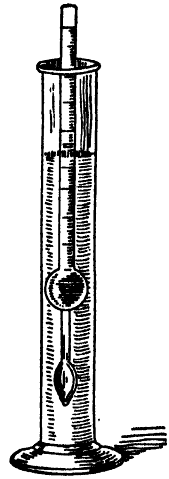


FIG. 3.

Method and Manipulation.—Prepare two solutions of such densities as will give a reading near each end of the scale when the hydrometer is immersed in them. Let r_1 and r_2 be the two readings obtained on the scale of equal divisions. Determine by means of a Mohr's balance the density ρ_1 and ρ_2 of the two solutions. The approximate method of Exp. 3 may be used. Determine also the mass m of the hydrometer to 0.1-g. Then, since by Archimede's principle m/ρ_1 and m/ρ_2 are the volumes of the liquid displaced in each case, the volume of the stem between r_1 and r_2 is $\frac{m}{\delta_1} - \frac{m}{\delta_2}$, and if $\rho_1 < \rho_2$, the volume per division is,

$$k = \frac{m}{r_2 - r_1}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right).$$

From this expression determine the value of k in cubic centimeters.

Solving the preceding equation for ρ_2, and dropping the subscript in the case of ρ_2 and r_2, so as to make them general, we obtain an equation that may be used in computing the density corresponding to any point r on the scale, *viz.*,

$$\rho = \frac{m\rho_1}{m - k\rho_1(r - r_1)}.$$

Compute ρ carefully for three or four values of r so chosen that, taken together with the two points determined by the experimental values, the whole range of the hydrometer will be covered, and plot these values as ordinates and the scale divisions as abscissas.

If a Baumé hydrometer is used, compare the values thus obtained with the attached direct-reading scale.

Caution.—Wash and dry the glass jar used with the Mohr's balance and the hydrometer itself after use with each solution, to avoid dilution or contamination of the standard solutions.

EXPERIMENT 5

SURFACE TENSION

If h is the height to which a liquid rises in a capillary tube, T the surface tension in dynes per centimeter length of film, and r the radius of the tube, then

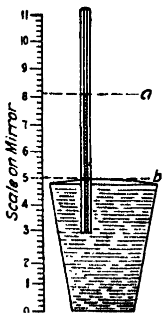


FIG. 4.

$$T = \frac{rh\rho g}{2 \cos \alpha}.$$

For water, since the density is (approximately) 1-g/cc, and the angle of contact on a clean glass surface is zero,

$$T = \frac{rhg}{2}.$$

Apparatus.—A glass tube; distilled water; a mirrored scale; a microscope with stage micrometer; cleaning solutions.

Manipulation.—After thoroughly cleaning the glass tube as directed below, heat it in a Bunsen flame until it becomes very pliable; then draw it into a capillary tube of about 0.5 mm diameter. In order to make the cross section of the capillary portion circular, the tube must be heated evenly on all sides. Hold it horizontally with both hands, thumbs uppermost, and rotate it between each thumb and forefinger while heating. Withdraw it from the flame just before drawing out, and main-

tain a tension on the tube until it has cooled. Break the tube so as to obtain a portion about 15 cm long, and clamp it vertically so that it extends into a vessel filled to the brim with the liquid. Obtain the height h by taking the reading a on the scale for the surface of the liquid in the tube and likewise the reading b for the surface of the liquid in the vessel.

The diameter $2r$ of the capillary tube is measured by means of the stage micrometer. Cut the tube at the point of the upper water meniscus. To cut the tube at a desired point mark it *lightly* with a file, and pull apart. Attach the tube to the stage by placing the piece to be measured in a slip of cork, with the cut end uppermost, and focus the microscope so that the end is seen clearly. When the cross hair is tangent to the bore of the tube and perpendicular to the direction of motions of the microscope, read the micrometer; then move the stage so as to make the cross hair tangent to the other side; and again read. Avoid backlash, due to looseness of the micrometer screw, by always moving the microscope in the same direction up to the desired position. Measure two diameters at right angles to each other. Use the average of the two. From height h and the diameter $2r$ compute the surface tension T.

Precautions.—The tubes will be found standing in a solution of chromic acid. Rinse with distilled water, dip in dilute nitric acid, rinse again with distilled water, and dry by shaking. The tube need not be completely dry when heated for drawing, provided it is kept horizontal until cold, so that drops of water will not run down upon the hot glass, causing it to crack.

An alternative method is to use capillary tubes about 15 cm long obtained from commercial stock of glass tubing. These are cleaned as directed above under Precautions. The diameter is measured as directed above by focusing the microscope on the end of the tube.

EXPERIMENT 6

TO MEASURE THE SURFACE TENSION OF A LIQUID BY MEANS OF THE DU NOÜY TORSION BALANCE

Apparatus.—The torsion balance consists of a wire which may be twisted through angles which are indicated by an index moving over a graduated disk. The sustaining force is applied

to the liquid film through an arm clamped to the middle of the wire and at right angles to it. The ring or yoke by means of which the liquid film is supported is suspended from the outer end of the arm.

Zero Adjustment.—Turn the wire by means of the worm adjustment at the index end until the index is at zero on the

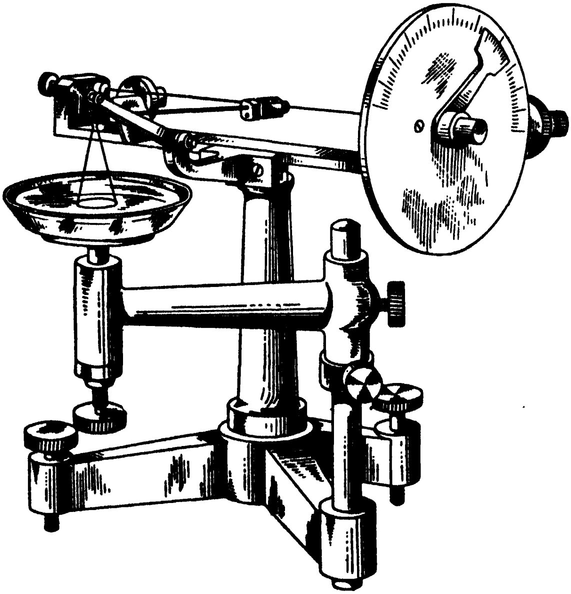


FIG. 5.

disk, the arm being free. Then turn the worm at the other end of the wire until the arm just clears the lower limit of the guide near the center of the arm. The ring or yoke must be suspended freely in the air from the outer end of the horizontal arm during these adjustments.

Calibration.—For the absolute determination it is necessary to calibrate the wire. This is accomplished as follows: Adjust for zero as directed above, the ring or yoke hanging freely in the air. Add a known mass (about 0.1-g) to the ring or yoke. As a support for the mass in the case of the ring place a small piece of paper of known mass over the ring. Turn the index until the arm just clears the lower limit of the guide, as in zero adjustment.

Note the reading of the index. Continue in this manner, adding additional masses until five or six readings differing by about 10° have been obtained. Plot the masses in grams as ordinates and the readings of index as abscissas. From the curve thus obtained the mass corresponding to any angle within the range represented may be obtained.

Manipulation in Case of Ring.—Adjust for zero as directed above when the ring hangs suspended in air. The table supporting the liquid is raised by turning the knob beneath until the ring is in the plane of the liquid surface and the contact between the ring and liquid is complete. Then turn the index end of the wire slowly until the ring snaps away from the liquid. The reading R on the disk for the position of the index is noted. Obtain from the calibration graph the mass M in grams to which R corresponds. The value of the surface tension is

$$T = \frac{Mg}{2\pi r},$$

where r is the radius of the ring in centimeters and g is the local value of the acceleration due to gravity in cm/sec^2.

Manipulation in Case of Yoke.—Adjust for zero as directed above when the yoke hangs suspended in air but with the vertical wires projecting into the liquid until the horizontal part is 1 cm above the liquid surface. Raise the liquid until the yoke is immersed. Then lower the liquid until the horizontal part of the yoke is about 1 cm above the liquid surface. Turn the index until the arm is at the zero position, and determine the index reading R. Obtain from the calibration graph the mass M in grams to which R corresponds. The surface tension then is

$$T = \frac{Mg}{S},$$

when S is the length of the yoke.

Precautions.—Care must be exercised to have the liquid pure. The platinum ring or yoke should be cleaned by heating to incandescence in a bunsen flame or washing in a 1 per cent solution of potassium dichromate in sulphuric acid and carefully rinsing with pure water. The dish for containing the liquid should be washed in a similar manner.

EXPERIMENT 7

TO DETERMINE THE COEFFICIENT OF VISCOSITY OF A LIQUID

The quantity to be determined is the force in dynes per square centimeter necessary to move a liquid area, with a velocity of one centimeter per second relative to a parallel plane in the liquid which is at a distance of one centimeter.

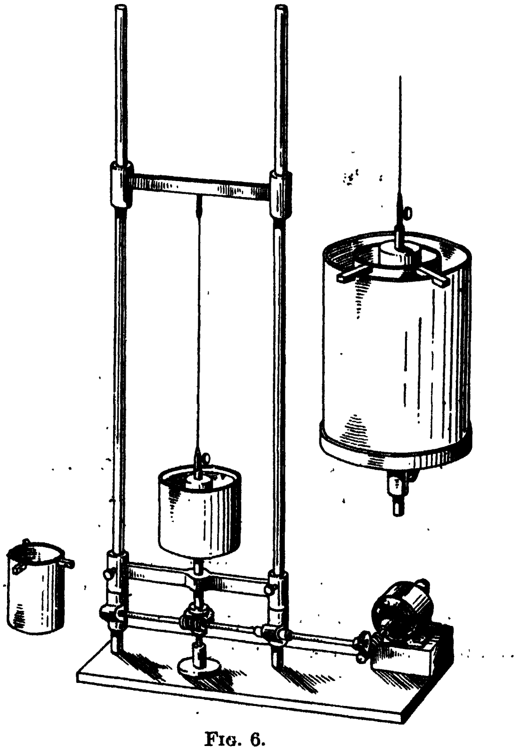


FIG. 6.

Apparatus.—A metal cylinder is suspended axially, by means of a torsion wire, in the liquid the coefficient of which is desired. The liquid is rotated uniformly by rotating the vessel that contains it. This is accomplished by means of an electric motor, connected to a speed reducer which permits various speeds from 10 to 60 r.p.m. A telescope and scale are provided for measuring the angle of torsion of the wire.

Method and Manipulation.—When the cylinder is suspended freely in air, set it into vibration as a torsion pendulum, and determine the time of one vibration T. By means of T and the moment of inertia of the cylinder, as computed from its dimensions, determine the moment of torsion K of the wire (see Exp. 17, Chap. I).

Add the liquid until the top of the cylinder is about 2 cm below the surface. Adjust the telescope and scale so that the zero of the scale is on the wire of the eyepiece. Gradually increase the speed until a deflection that can be determined with sufficient accuracy is obtained. By means of a stop watch determine the number of revolutions per second n.

The torque due to the viscosity can be shown to be

$$8\pi^2 \frac{R_1^2 R_2^2 n S}{R_2^2 - R_1^2} \eta,$$

where R_1 and S are the radius and length of the cylinder and R_2 the radius of the vessel containing the liquid.

As this torque must equal the elastic restoring torque $K\theta$ due to the wire, we have

$$K\theta = 8\pi^2 \frac{R_1^2 R_2^2 n S}{R_2^2 - R_1^2} \eta.$$

The coefficient of viscosity therefore is

$$\eta = \frac{(R_2^2 - R_1^2)K\theta}{8\pi^2 R_1^2 R_2^2 S n}$$

or

$$\eta = k\frac{K\theta}{nS},$$

where $k = (R_2^2 - R_1^2)/8\pi^2 R_1^2 R_2^2$.

The foregoing neglects the forces acting upon the ends of the cylinder. To correct for this the experiment is repeated using the same wire and a cylinder of the same radius but of different length. If L is the viscosity torque acting upon the ends of the cylinder, then in the case of each cylinder we have, using the same wire,

$$K\theta_1 = \frac{S_1 n_1}{k}\eta + L;$$

$$K\theta_2 = \frac{S_2 n_2 \eta}{k} + L.$$

Subtracting gives

$$K(\theta_1 - \theta_2) = \frac{\eta}{k}(S_1n_1 - S_2n_2);$$

$$\therefore\ \eta = kK\left(\frac{\theta_1 - \theta_2}{S_1n_1 - S_2n_2}\right).$$

Compute the value of k from the dimensions of the apparatus as indicated above, and determine S_1S_2, n_1n_2, $\theta_1\theta_2$ by performing the experiment once with each cylinder.

If the speed is kept constant, then $n_1 = n_2 = n$; and if $S_1 - S_2 = d$, then

$$\eta = kK\frac{(\theta_1 - \theta_2)}{nd}.$$

Instead of using two cylinders of different length as indicated above, the same cylinder may be used, and two different amounts of the liquid each of which will leave the top end of the cylinder exposed. If H_1 is the height of the liquid during one run, and H_2 the height during another run, then

$$d = H_1 - H_2,$$

and, as before,

$$\eta = kK\frac{(\theta_1 - \theta_2)}{nd}.$$

CHAPTER III

HEAT

EXPERIMENT 1

ATMOSPHERIC PRESSURE

The pressure of the atmosphere is measured by the weight of the column of pure mercury at 0°C. that it can support.

If H_0 is the height of the mercury column, the pressure in dynes per square centimeter is

$$H_0 \rho g,$$

where ρ is in the density of mercury (13.596).

If the temperature is t, and if the reading H for the height of the column is obtained by means of a scale that is correct at the temperature t', the true height

$$H_t = [1 + \alpha_1(t - t')]H,$$

where α_1 is the coefficient of expansion of the scale.

If the height of the mercury column is H_t when its temperature is t, its height at 0°

$$H_0 = \frac{H_t}{1 + \alpha_2 t},$$

where α_2 is the coefficient of expansion of mercury.

The height of the column including both corrections is

$$H_0 = \frac{1 + \alpha_1(t - t')}{1 + \alpha_2 t} H.$$

If the scale is correct at 62°F., and H and t are obtained in English and Fahrenheit units, the equation becomes

$$H_0 = \frac{[1 + \alpha_1(t - 62)]}{1 + \alpha_2(t - 32)} H,$$

where $\alpha_2 = 0.0001001$ and $\alpha_1 = 0.00001043$ both per degree Fahrenheit.

The tension of the mercury vapor in the vacuum above the mercury causes a small depression, the correction for which may be obtained from tables (see Table 23).

The downward pull due to the meniscus of the mercury also causes a depression. This is reduced by using a tube of large diameter (25 mm). The correction may be obtained from tables arranged for that purpose. Frequently the scale is shifted to offset this error.

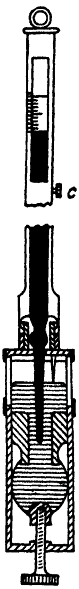


FIG. 1.

Apparatus.—Fortin's barometer. The construction of this barometer is as indicated in Fig. 1. The scale attached to the metal casing is adjusted so as to measure from the end of the ivory point projecting down from the roof of the cistern. The mercury surface in the cistern can be raised or lowered by means of the screw underneath. The scale is provided with a vernier, and on the opposite side of the tube from the vernier and connected to the vernier is a slide whose lower edge is on a level with that of the vernier. By means of the head *c* the vernier and slide may be moved up or down. A thermometer is attached to the metal casing of the barometer.

Manipulation.—Record the temperature, and adjust the surface of the mercury in the cistern so that it just touches the ivory point. Then adjust the vernier so that the top of the meniscus is in line with the lower edges of the vernier and slide. Record the reading for the height, and correct it for expansion of scale and mercury by means of the equation given above.

Obtain correction from the table arranged for the barometer, and compare with corrected height (see Table 22 of Appendix).

EXPERIMENT 2

CALIBRATION OF A THERMOMETER

The errors that this calibration is designed to correct are those due to the inaccuracies of the zero and boiling points on the thermometer scale and the irregularity of the bore of the tube. On account of

these errors the thermometer scale does not accurately represent temperatures.

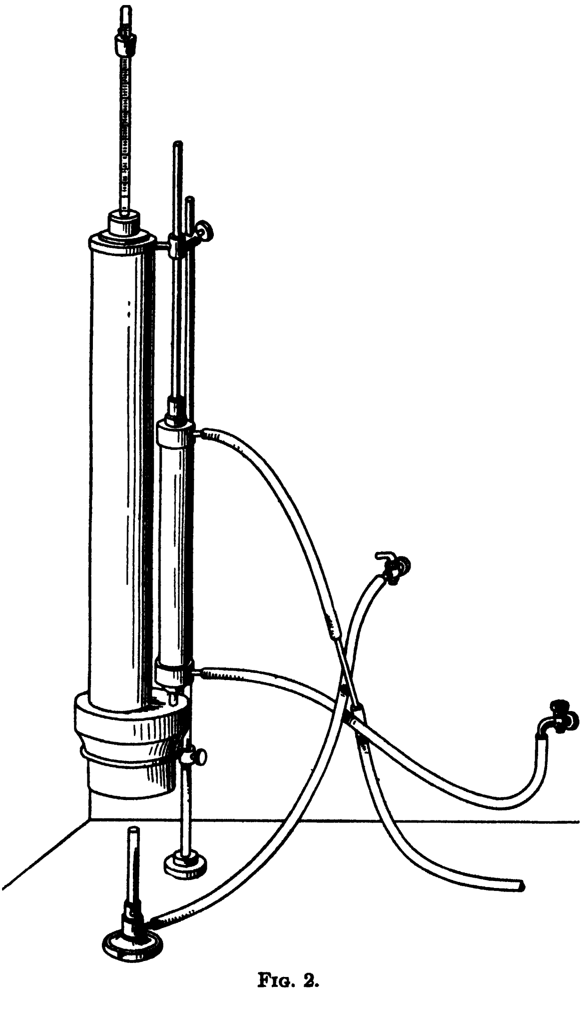


FIG. 2.

Apparatus.—Thermometer; barometer; funnel of finely divided ice; a boiler for holding the thermometer in steam.

Manipulation.—Immerse the thermometer up to the zero mark in melting ice. Set a small piece of mirror against the ther-

mometer tube, and note the reading on the scale when the end of the mercury thread and its image in the mirror are in line, in order to avoid parallax.

After the mercury becomes stationary, note the reading z on the thermometer scale, avoiding parallax by use of a mirror. The amount z represents the difference in degrees between the true zero temperature and the zero on the scale. Then place the thermometer in the boiler as shown in Fig. 2, so that the bulb is not less than 2 cm above the water surface. Allow the water to boil; and when the mercury surface becomes stationary, note the reading a on the scale. Determine the barometric height, and from it determine the true temperature $A°$ of the steam. The boiling temperature of water at 760 mm (29.921 in.) pressure is 100°C. It decreases 1°C. for about 26.7 mm (1.05 in.) decrease in pressure.

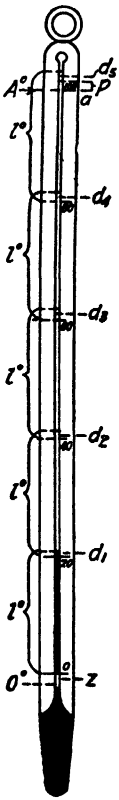


FIG. 3.

$$\therefore A° = 100° - \frac{760 - H}{26.7}.$$

$A°$ is the true temperature corresponding to the reading a on the scale when the thermometer was exposed to the steam.

Separate a thread of mercury which is about 20 degree divisions long. This is best done by holding the thermometer in a horizontal position and giving it a light, quick tap at the upper end. If the thread that separates is not of the proper length, join it to the rest, and note the position on the scale where they come together. This, owing to a small air bubble adhering to the glass, is in general the place where the mercury separates when the thermometer is again tapped even though mercury has been forced by the point. Then warm or cool the bulb until the desired length is above the joining point and tap. If unsuccessful, repeat the process. Having separated a thread of the desired length, place one end even with the zero on the scale, and read the fraction d_1 that the other end is above or below the 20 mark. Then place the lower end at 20, and read the difference d_2 at 40.

Continue thus until the differences d_3 at 60, d_4 at 80, and d_5 at 100 have been obtained. Repeat by placing the upper end even with 100 and reading the difference at 80, etc.

Consider the quantities as indicated in Fig. 3. Let

$$100 - a = p.$$

Let l° be the number of degrees that the temperature of the mercury must be raised to increase its volume an amount equal to the volume of the calibrating thread.

The markings on the thermometer tube are sufficiently accurate so that the difference between the length of one division on the scale and the actual length of expansion for 1° is so small that it may be neglected. Several divisions, however, do not represent the same number of degrees, for the differences accumulating make an appreciable error. Let the values obtained for the small distances z, p, and $d_1 \ . \ . \ . \ d_5$ represent degrees. Then the number of degrees necessary to expand the volume of the mercury by the volume of the tube from 0° to a is

$$z + l^\circ - d_1 + l^\circ - d_2 + l^\circ - d_3 + l^\circ - d_4 + l^\circ - d_5 - p,$$

or

$$5l^\circ + z - d_1 - d_2 - d_3 - d_4 - d_5 - p.$$

But, experimentally, it is known that A° expands the mercury from 0° point to a.

$$\therefore \ A^\circ = 5l^\circ + z - d_1 - d_2 - d_3 - d_4 - d_5 - p,$$

and

$$l^\circ = \frac{A^\circ - z + d_1 + d_2 + d_3 + d_4 + d_5 + p}{5};$$

i.e.,

$$l^\circ = \frac{A^\circ - z + \Sigma d + p}{5}.$$

As A°, z, Σd, and p are known, l° becomes known.

If z was above 0 on the scale, and if any value for d represents a fraction below 20, 40, 60, 80, or 100, then their signs in the equation for l and in the following equations must be changed.

The temperature for the division on the scale corresponding to the subscript of t is then as follows:

$$
\begin{aligned}
t_0 &= +z^\circ, \\
t_{20} &= z + l^\circ - d_1, \\
t_{40} &= z + 2l^\circ - d_1 - d_2, \\
t_{60} &= z + 3l^\circ - d_1 - d_2 - d_3, \\
t_{80} &= z + 4l^\circ - d_1 - d_2 - d_3 - d_4, \\
t_{100} &= z + 5l^\circ - d_1 - d_2 - d_3 - d_4 - d_5 \\
t_{100} &= A^\circ + p \text{ } (check), \\
t_1 &= z + \frac{t_{20} - z}{20} = z + k, \\
t_2 &= z + 2k, \\
t_3 &= z + 3k, \\
t_{20} &= z + 20k, \\
t_{21} &= t_{20} + \frac{t_{40} - t_{20}}{20} = t_{20} + k', \\
t_{22} &= t_{20} + 2k',
\end{aligned}
$$

etc.

Record the number of the thermometer used.

Compute the correction to be applied to the scale reading at 0, 20, 40, 60, 80, and 100, and plot a correction curve with the corrections as ordinates and scale readings as abscissas. The smoothest curve that can be drawn so as to pass close to all the plotted points is the correction curve required. The thermometer calibrated is to be used in the heat experiments which follow.

EXPOSED-STEM CORRECTION

If it is desired to correct the temperature reading for the fact that a large part of the thermometer stem is at a lower temperature than the bulb, an auxiliary thermometer must be supported beside the first so that its bulb is at the same level as the center of the exposed portion of the mercury thread in the first. If

R_1 = the reading of the thermometer,
R_2 = the point on its scale to which it is immersed,
t° = the reading on the auxiliary thermometer,
β = the coefficient of volume expansion of mercury (0.0001813 per °C.),
α = the coefficient of linear expansion of glass (0.0000085 per °C.),
t_c = the corrected temperature desired,

then

$$t_c = R_1 + (R_1 - R_2)(R_1 - t^\circ)(\beta - 3\alpha).$$

EXPERIMENT 3

COEFFICIENT OF LINEAR EXPANSION OF A SOLID

Apparatus.—The solid is in the form of a tube about 1 m long and 0.8 cm in diameter. This tube is placed inside another tube of about 5 cm diameter which is packed in felt. Near each end of the small tube is a collar from which a bone rod projects at right angles and extends just through the side of the outer tube. At each end of the small tube is a trap containing a thermo-

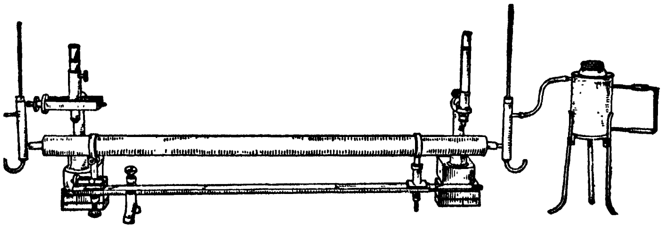


FIG. 4.

meter. A heavy iron block at each end supports the tube and a telescope. One of the telescopes is provided with a stage micrometer. One of the supports for the tube carries a screw adjustment so that the tube may be moved in the direction of its length.

Method and Manipulation.—Pass a stream of ice water through the small tube. By means of a steel scale or tape measure the distance S_1 between the fine index lines on the bone projections. Focus the microscopes, and adjust until the cross hairs are over the index lines. Be careful to avoid backlash on the part of the micrometer screw by approaching the index line in the direction in which the tube will expand. Obtain the micrometer reading R_1 and the readings t_1' and t_2' of the thermometers. The thermometer readings t_1 and t_2 must be properly corrected. Their average will give the initial temperature t_1 of the tube. Pass steam through the small tube, and when further expansion has ceased move the tubes by means of the screw support until the index line coincides with the cross hair of the telescope which is not provided with the micrometer. Move the other telescope by

turning the micrometer screw until the index line and the cross hair coincide. Again obtain the micrometer reading R_2 and the temperatures t_3' and t_4' from which t_2 is obtained as above.

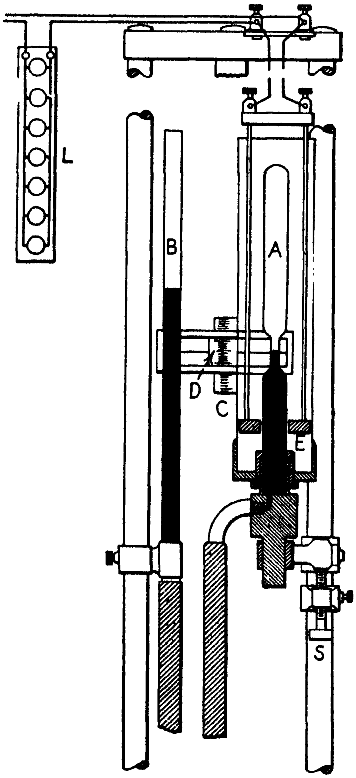


Fig. 5.

The coefficient of expansion

$$\alpha = \frac{R_2 - R_1}{S_1(t_2 - t_1)}.$$

EXPERIMENT 4

THE GAS LAW. BOYLE'S AND CHARLES'S LAWS COMBINED

Apparatus.—The gas is enclosed in a glass tube, as shown at A (Fig. 5). The mercury in contact with the gas in A communicates through a rubber tube with the mercury contained in a parallel glass tube B. The glass tube A is surrounded by a glass jacket filled with kerosene which may be heated by passing an alternating electric current through a coil of five turns of No. 24 Iala wire wound on the stirrer E. The current may be obtained from the house circuit, using a bank of incandescent lamps L connected in series with the coil of the stirrer. As the lamps in the bank are in parallel, the current may be altered by connecting or disconnecting the various lamps. All parts in contact with mercury and liquid are of glass. The apparatus is supported by two parallel vertical rods 1.5 m in height and having a tripod base. A microscope clamped to one of the vertical rods may be focused on the mercury surface in A. The eyepiece of the microscope is provided with a cross hair. The gas used in A is dry hydrogen.

Readings.—The volume of the gas is kept constant by adjusting the screw S until the mercury surface in A is even with the

cross hair in the microscope. The pressure in centimeters of mercury height is the barometer reading plus the difference in heights of the mercury surface in A and in B. This difference is obtained from the scale C. The slide D carries a vernier and a mirror upon which is an index line which may be placed, free from parallax, even with the mercury surfaces. The temperature is obtained from a thermometer suspended in the liquid surrounding the tube A.

Method and Manipulation.—When Boyle's law and Charles's law are combined, the relation $PV/T = R$ (a constant) is obtained, where T is the absolute temperature, P the pressure, and V the volume. When the gas in A is at the room temperature, determine its pressure, as directed above, and the absolute temperature. Raise the temperature about 5° by passing a current through the coil, being careful to adjust the current so that the temperature is constant while the readings are taken. Adjust, and determine again the pressure and absolute temperature of the gas. Continue thus until several sets have been obtained.

Plot the temperatures as abscissas and the pressures as ordinates. Extend the graph until it intersects the axis of abscissas. Note the significance of the intercept.

EXPERIMENT 5

THE RADIATING POWER OF DIFFERENT SURFACES

The quantity to be determined is the number of calories lost per second, through radiation, by a unit area of a radiating surface when its temperature is one degree above the temperature of the surroundings.

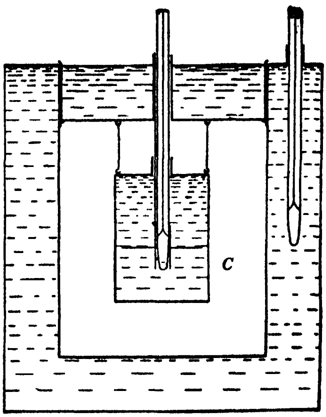


Fig. 6.

Apparatus.—The radiating body consists of a bright or black vessel C filled with water. This vessel is suspended or supported on an insulating base so as to be completely surrounded by a water jacket, as shown in Fig. 6. The radiator C and jacket are provided with stirrers. The temperature of the radiating

body and that of the surrounding surface are obtained by means of two thermometers placed as shown in the figure.

Thermometers reading 0.1° should be used.

Method and Manipulation.—Determine the mass and area of the vessel C, and fill it with water at about 60°C. Observe the temperatures of the water in the vessel C and of the water in the surrounding jacket at the end of every 2 minutes during a period of 20 minutes. In order to obtain a uniform temperature, the water in C and in the jacket must be stirred constantly. Always

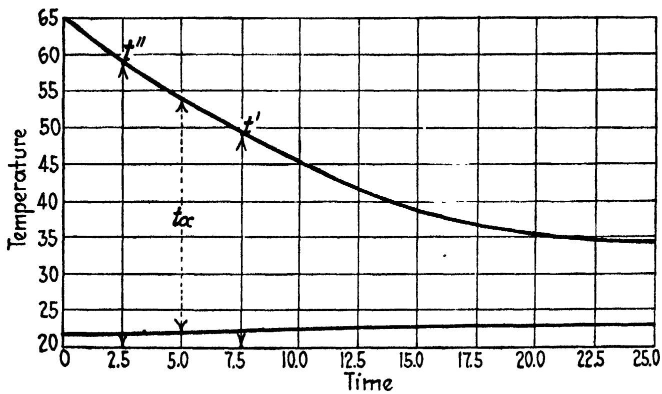


FIG. 7.

tap a thermometer that is indicating a decreasing temperature just before reading. Obtain the radiation curve by plotting the temperatures as ordinates and times as abscissas. Likewise, on the same sheet and with the same units obtain the curve showing the change in the temperature of the surrounding jacket with time. The scale of ordinates should be as large as the range of temperatures to be plotted in each case permits. In the following manner compute the coefficient of radiation at the instants 5, 10, 15 minutes: subtract the temperature t' at 7.5 minutes from the temperature t'' at 2.5 minutes as obtained from the radiation curve. Using this value for $(t'' - t')$, compute the radiating power k from the relation

$$k = \frac{(M + c + e)(t'' - t')}{TAt_a},$$

where M = the mass of water in the radiating vessel,
c = the thermal capacity of the vessel,
e = the thermal capacity of the thermometer, approximately equal to 1 cal,
t_a = the difference in temperature between the radiator and jacket at the instant selected for computing k,
A = the area in sq cm of the radiating vessel,
T = the time in seconds, *i.e.*, 300.

Show the variation of these coefficients by plotting them as ordinates, using the corresponding values of t_a as abscissas (Fig. 7).

Make the foregoing determinations in the case of a bright and a black radiating vessel.

EXPERIMENT 6

THE ABSORBING POWER OF DIFFERENT SURFACES

The quantity to be determined is the number of calories gained per second through radiation by a unit area of an absorbing surface when its temperature is one degree below the temperature of the surroundings.

Apparatus.—Same as in Exp. 5.

Manipulation.—Fill the radiating vessel C with water at nearly 0°C., and proceed in the same manner as in Exp. 5.

EXPERIMENT 7

SPECIFIC HEAT

The quantity to be determined is the number of calories necessary to raise the temperature of one gram of a given substance one degree.

Apparatus.—The calorimeter C surrounded by a water jacket B; a heater A consisting of a central metal tube surrounded by a heating coil embedded in insulating material which forms a heat-insulating jacket. The material, the specific heat of which is desired, may be suspended inside the central tube. Two thermometers; a thermocouple and galvanometer; a watch.

Method and Manipulation.—Obtain the mass of the solid to be used, and suspend it by means of a string inside the inner tube

of the heater. Pass an electric current through the heater coil until the solid assumes the equilibrium temperature of the heater. Obtain the mass of the calorimeter, and fill it with water at room temperature to a point about 2 cm from the top. Obtain the mass of both water and calorimeter.

Stir the water in the calorimeter constantly, and observe the temperature of the water in the calorimeter at the end of every minute for a period of about 8 minutes. If a thermometer is used, this is done by observing the position of the mercury surface on the scale of the thermometer, being careful to avoid parallax, by the use of a mirror or a stem microscope. If a thermocouple is used, read the deflection on the galvanometer scale, one junction being in the water and the other in the thermos bottle containing the liquid of reference temperature. Immediately after the last reading, pass the solid quickly into the calorimeter by means of the attached string. Stir, and record the temperature at the end of every 30 seconds until the temperature of the water has reached a maximum; and after that continue recording the temperature at the end of every minute for an interval of about 10 minutes. During the experiment the temperature of the water jacket of the calorimeter must be recorded at short intervals. Record the temperature t_b inside the heater.

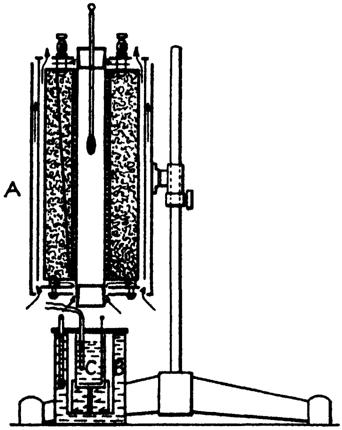


FIG. 8.

Correction for Heat Loss.—To obtain the corrected rise in the temperature of the water in the calorimeter, plot the temperature readings on coordinate paper, using times as abscissas and temperatures as ordinates.

This curve has three regions *AB*, *BC*, and *CD*, *A* being the initial temperature of the water in the calorimeter. It is obvious that the temperatures represented by points on the portion *BCD* if above the temperature of the water jacket are low owing to heat losses by radiation, conduction, etc., and must be corrected in

order to find the true rise in temperature of the water. These corrections may be obtained in the following manner:

From the portion *CD* (see Fig. 9) obtain the fall in temperature per second *k* by dividing the fall in temperature in an interval *ab* of two or three minutes by the number of seconds in the interval. This may be taken as the rate of fall in temperature when the calorimeter is at the temperature t_a corresponding to a point at the middle of the interval *ab* used. From a point *P* corresponding to t_a on the axis of ordinates draw a line parallel to the axis of abscissas, and on it let a convenient length represent the value of

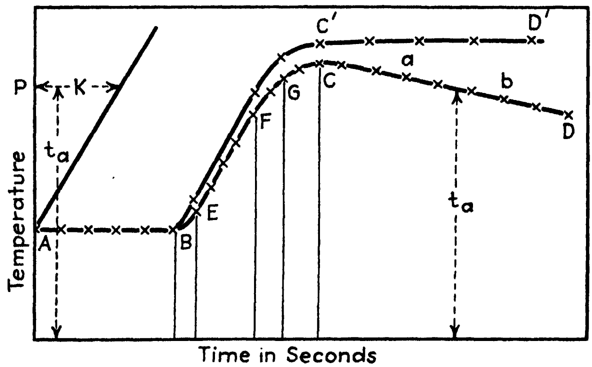


Fig. 9.

k. From the point *A* on the temperature axis representing the temperature t_j of the water jacket, at the time corresponding to the time of t_a, draw a straight sloping line through the extremity of line *k*. From this line the rates *k* corresponding to any temperature point on *BC* may be obtained, being simply, by Newton's law of cooling, the horizontal distance to the temperature axis. Divide *BC* into segments over each of which the curve may be considered approximately straight. These segments need not be of equal lengths. Obtain the rate k_1 for the temperature at the mid-point of each segment *BE*. Multiply k_1 by the time interval T_1 of the segment in seconds. The amount k_1T_1 gives the loss of temperature during the segment due to radiation, etc. Proceed in this manner in the case of each segment. The total correction to be added to the observed temperature at the end of a segment is the sum of the k_1T_1

correction for the lower segments including that for the segment in question. Continue in this manner in connection with all the segments. Add the total corrections graphically to the corresponding temperatures on the curve, and obtain thus the corrected temperature curve.

It is obvious that if the work has been correctly performed, the upper part of the curve, *viz.*, $C'D'$, will be parallel to the axis of abscissas and represents the temperature t_m that the water would have attained had there been no heat losses. The initial temperature t_B may be taken as that of the point of intersection of AB and EF produced. The rise in the temperature of the water due to the heat obtained from the solid is therefore $t_m - t_B$.

It is to be noted that if the lower part of the curve is below the temperatures of the water jacket, the corrections are negative for the part of the curve that is below. It is desirable in this method to have the initial temperature of the water the same as the temperature of the water jacket. If the temperature of the water jacket is not constant, care must be taken, in finding k_1, to use the difference between the temperature of the mid-point on the segment and that of the water jacket at the same instant.

If M = the mass of the water,
m = the mass of the solid,
t_B = the initial temperature of the water,
t_m = the final temperature of the water,
t_h = the temperature of the steam or solid,
c = the water equivalent of the calorimeter plus stirrer,
e = the water equivalent of the thermometer,
then the specific heat

$$S = \frac{(M + c + e)(t_m - t_B)}{m(t_h - t_m)}.$$

From the values obtained for these quantities, compute the specific heat of the solid.

If the thermocouple is substituted for the thermometer in the calorimeter, as directed in the following experiment, the factor e in the preceding equation may be neglected. A short piece of rubber tubing should cover the handle of the stirrer in order to reduce thermal contact between the hand and stirrer.

EXPERIMENT 8

CALIBRATION OF THERMOCOUPLE FOR CALORIMETRIC DETERMINATIONS

The use of the thermocouple for measuring temperatures has, when properly arranged, many advantages over the thermometer. The thermocouple depends on the principle that, if two dissimilar metals or alloys in the form of wires are arranged in circuit with a galvanometer *G* and a regulating resistance *R* (Fig. 10) so as to form a pair of junctions *A* and *B*, an electric current is produced if one junction (*i.e.*, *A*) is kept at a constant temperature (0° or room temperature) and the other is raised or lowered in temperature. The galvanometer *G* indicates the amount of current; and if the proper pair of dissimilar metals are employed (*i.e.*, copper and Advance wire), the galvanometer deflections will be practically proportional to the temperature difference between the two junctions.

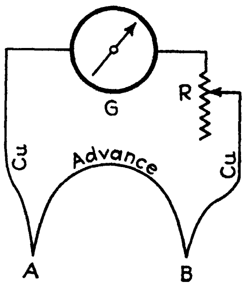


Fig. 10.

To maintain junction *A* at a constant temperature it may be anchored in paraffin inside a glass protection tube which is inserted in a thermos bottle containing fine ice (0°C.) or water at room temperature. To obtain the zero point of the galvanometer first place junction *B* also in the thermos bottle. Thus with no temperature difference, no current will flow in the circuit. Good insulation is necessary. Enameled No. 25 Cu. wire and No. 25 Advance wire are used. The wires are insulated and placed inside a small-sized brass tube, the junction being soldered to the tube at one end, and the wires well taped at the other. Guard against all possibilities of short circuits.

To calibrate this thermocouple take a large calorimeter containing a stirrer. The junction *B* may now be taken out of the thermos bottle and placed in water contained in a calorimeter. The water in the calorimeter may be taken 25° or 30° above the temperature in the thermos bottle. The resistance *R* is adjusted so that the deflection on the galvanometer will come near the end of the scale. (50-cm. scale.) Use a good tenth-degree thermome-

ter to measure temperatures in the calorimeter. Now cool the water by steps of 5° or 6° by adding a proper amount of colder water each time, reading the temperature on the thermometer in the calorimeter and the corresponding deflection of the galvanometer after thoroughly stirring the water in the calorimeter. Taking the highest temperature first prevents going beyond the limits of the scale for the temperature range desired.

To plot the calibration curve, take galvanometer deflections as the abscissas and the differences of temperature between the thermometer in the thermos bottle and the one in the calorimeter as ordinates. Using temperature differences along the ordinate, this value when added to the temperature of the water in the thermos bottle will always give the true temperature of the hot junction in the calorimeter even though the thermos-bottle temperature may be different from time to time. For specific heat and heat of vaporization determinations, the water in the thermos bottle may be kept at room temperature. For heat of fusion it is best to use fine ice in the thermos bottle (0°C.). The same calibration curve will serve in all cases. The water in the calorimeter for these experiments is taken very little above room temperature, and the radiation curve obtained as directed in Exp. 7. The thermocouple method for measuring temperature changes will be found very accurate and easy to follow, especially if the lamp and scale method for determining galvanometer deflections is used. It is most important that there be thorough stirring in all cases.

In the calibration of the thermocouple, steady agreement between the thermometer and the thermocouple can be quite well ascertained by their drifting along together. Parallax in reading the thermometers should be avoided, and when necessary, exposed stem correction should be made (see Exp. 2). Plot all curves on large-size cross-section paper (20 in.) so that readings may be as accurately obtained from the curve as from the instrument (see sample curve on bulletin board).

EXPERIMENT 9

LATENT HEAT OF FUSION

The quantity to be determined is the number of calories necessary to change one gram of ice to water.

Apparatus.—A calorimeter surrounded by water jacket; one thermometer; a watch.

Method and Manipulation.—Obtain the mass of the calorimeter, and fill it about two-thirds or three-fourths full of water at room temperature. Obtain the mass of the water and calorimeter, stir constantly, and record the temperature of the water at the end of every minute for a period of about 5 minutes. Record the time of day of the first reading. Then, quickly, after the last reading, drop three pieces of dry ice each about the size of an egg totaling about 130 cc volume into the calorimeter. To insure the dryness of the ice, keep it wrapped in cloth or absorbent paper until introducing it into the calorimeter. Noting the time of day of the first reading, record the temperature at the end of every 30 seconds until the ice has melted; then continue taking the temperature at the end of every minute for an interval of 8 minutes. Likewise at the same time obtain the temperature of the water jacket. Continue stirring throughout the entire series of readings. Plot these readings, and from the curve determine the fall in temperature of the water by the construction explained under Exp. 7. Weigh the calorimeter and water. The mass of the ice introduced is the difference between this and the mass previously obtained.

If M = the mass of the water,
m = the mass of the ice,
t = the initial temperature of the water,
t' = the final temperature of the water,
e = the water equivalent of the thermometer,
c = the water equivalent of the calorimeter plus stirrer,
then the latent heat of fusion

$$L = \frac{(M + c + e)(t - t') - mt'}{m}.$$

If the thermocouple is used in place of the thermometer, proceed as directed in Exp. 8.

EXPERIMENT 10

LATENT HEAT OF VAPORIZATION

The quantity to be determined is the number of calories necessary to vaporize one gram of water.

Apparatus.—The apparatus is arranged as shown in Fig. 11. Steam is passed from the boiler into the trap and from there down

into the condenser immersed in the water in the calorimeter where the steam is condensed, and imparts its heat to the condenser and surrounding water.

Method and Manipulation.—Obtain the mass of the calorimeter and of the condenser. Fill the calorimeter with water at room temperature, sufficient to cover the condenser. Obtain the mass of the water and condenser and connect condenser to boiler, as shown in Fig. 11. Connection with the boiler is not made until the instant the steam is to be admitted. Stir the water in the calorimeter, and record its temperature at the end of every minute during a period of about 8 minutes, noting the time of day of the first reading. Immediately after the last reading allow the steam to enter the condenser by closing the stopper at the top. Stir, and record the temperature at the end of every 30 seconds; and when the temperature of the water has increased about 10°, disconnect the boiler by opening the stopper. Continue recording the temperature at the end of every minute for a period of about 8 minutes. All the foregoing time readings must be continuous by the clock. The temperature of the water jacket must be recorded at frequent intervals. Wipe the condenser dry, and determine its mass including mass of the steam condensed. From the barometric pressure determine the temperature of the steam (see Table 21). Plot the temperature readings as ordinates and the corresponding times as abscissas. Obtain the rise in temperature by the method explained in Exp. 7.

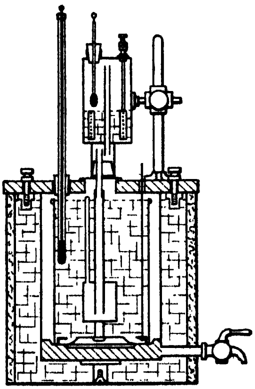


FIG. 11.

If M = the mass of the water,

M_1 = the mass of the calorimeter,

m = the mass of the condenser,

m' = the mass of the steam condensed,

t = the initial temperature of the water,

t' = the final temperature of the water,

t_s = the temperature of the steam,

s = the specific heat of brass,

e = the water equivalent of the thermometer,

then the latent heat of vaporization

$$L = \frac{(M + M_1 s + ms + e)(t' - t) - m'(t_s - t')}{m'}.$$

Precautions.—The condenser must be dried before using by passing a current of warm air through it.

If the thermocouple is substituted for the thermometer, proceed as directed in Exp. 8. In this case e may be neglected.

EXPERIMENT 11

TO DETERMINE THE VAPOR PRESSURE OF WATER AT DIFFERENT TEMPERATURES

Apparatus.—An airtight boiler provided with a condenser. A tube leads from the condenser to a large vessel V containing air, the purpose of which is to render the pressure steady. The vessel V is connected to a manometer for determining the pressure. Pressures less than one atmosphere are obtained by exhausting the air from the boiler and vessel V by means of a filter pump connected to the tube F. The bulb of the thermometer is placed in oil contained in a closed tube extending into the boiler.

Method and Manipulation.—Obtain the temperature of boiling when the pressure is one atmosphere; then start the exhaust pump, and reduce the pressure about 10 cm. Turn the stopcock C so as to close the tube leading to the boiler. When the mercury in the thermometer becomes stationary, read the temperature and pressure. In case the apparatus is not quite airtight, the pressure will rise slowly after the pump is cut off. The minimum temperature should be taken, and the manometer readings obtained at the same instant. The barometer must be read in order to compute the resultant pressure on the vapor in the boiler. This pressure should not be corrected for temperature unless the manometer difference is also corrected, and the latter is not measurable accurately enough to make this advisable. Then open C, and again reduce the pressure 10 cm. Close C, and when the thermometer becomes constant obtain the temperature and pressure. Continue thus until the pressure is as small as possible. Under the best conditions it is possible to have the water boil at a temperature less than 30°C.

Obtain the vapor-pressure curve by plotting the pressures and the corresponding temperatures on coordinate paper, the temperatures being laid off along the axis of abscissas.

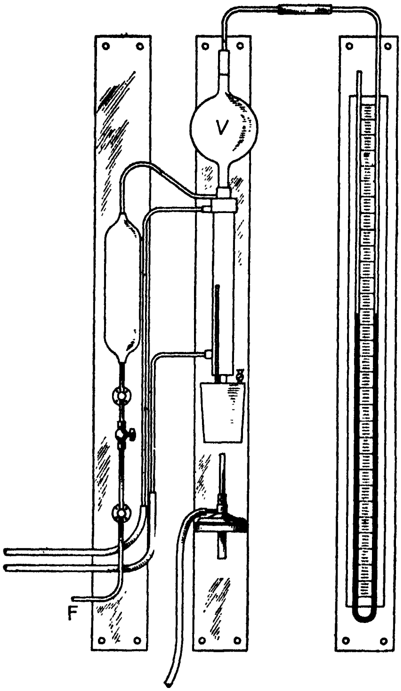


FIG. 12.

EXPERIMENT 12

TO DETERMINE THE RELATIVE HUMIDITY OF THE ATMOSPHERE

The quantity to be determined is the ratio of the mass of the water vapor contained in a unit volume of air to the mass of water vapor it would contain if the air were saturated at that temperature.

1. THE DEW-POINT METHOD

Apparatus.—The dew point, or Daniell's, hygrometer consists of two glass bulbs joined by a glass tube as shown in the figure. The lower bulb has a bright metallic surface which enables the formation of dew to be more readily detected. This bulb also contains a thermometer. A thermometer for determining the

room temperature is attached to the supporting stand. The instrument contains a quantity of ether sufficient nearly to fill one of the bulbs, no air being present.

Method and Manipulation.—In a vessel which is well surrounded by felt place some water and finely divided ice, and in this immerse the upper bulb after the ether has been collected into the lower bulb. Record the air temperature t_a, and, at the instant that dew begins to form on the bright surface of the lower bulb, record the temperature t_s of the metallic surface as indicated by the enclosed thermometer. Remove the ice and water, and note the temperature t_s at the instant that the dew disappears. If the cooling is stopped as soon as the dew begins to form, the two values of t_s will not differ by more than a degree centigrade. Repeat several times, and obtain the average temperature t_s. This gives the temperature at which the amount of vapor present in the free air is sufficient to saturate air at the pressure of the free air.

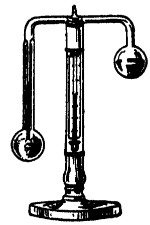


FIG. 13.

Since the vapor present is in the free air, its pressure does not appreciably change when its temperature is lowered. The pressure p_d of the vapor present in the air is, therefore, the same as the saturation vapor pressure of water at the dew-point temperature and may be determined from the table of vapor pressures given in Table 24. The saturation pressure p_s corresponding to the room temperature is obtained from the same table. From the gas law $pv = RmT$ we have the law that the pressures are to each other as the masses. Therefore the relative humidity is

$$H = \frac{m}{m_s} = \frac{p_d}{p_s}.$$

From the values p_d and p_s calculate H.

2. THE WET AND DRY BULB, OR THE PSYCHROMETER METHOD

Apparatus.—Two thermometers are mounted side by side. The bulb of one is covered with a cloth which is kept saturated with water.

Manipulation.—By swinging the instrument about the handle, pass a current of air over the two bulbs of the thermometers

when they are dry, and obtain the readings of the thermometers. Their difference gives the thermometer correction. Half of the correction is to be added to all readings taken with the thermometer reading the lower under these conditions, and half to be subtracted from all readings taken with the other. Then saturate the cloth with water, and again pass a current of air over the bulbs. Obtain the thermometer readings when they become

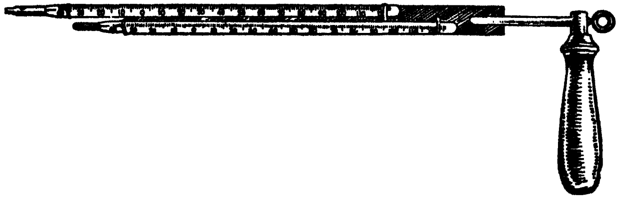


FIG. 14.

steady and the barometer reading B. Correct the readings, and from the difference in the two thermometer readings, the reading t_1 of the wet-bulb thermometer, and the barometric pressure obtain the vapor pressure p_d from Table 25.

From Table 24 obtain the vapor pressure p_s corresponding to the reading of the dry-bulb thermometer. From the values p_d and p_s obtain the humidity H as above.

A direct-reading instrument on the psychrometer principle called a *hygrodeik* is also provided. Read the directions printed upon it, take readings on the two thermometers, and use the chart and swinging arm to find the relative humidity, the absolute humidity, and the dew point. The instrument gives values that are too high unless placed in a strong current of air.

EXPERIMENT 13

THE MECHANICAL EQUIVALENT OF HEAT

The quantity to be determined is the number of ergs equivalent to the energy of one calorie.

Apparatus.—A conical cup a (Fig. 15) is placed on ebonite supports inside a larger cup which is mounted on a vertical axis. A friction cone b fits the cup a and supports a flanged wheel c, above which is a heavy metal ring d which serves as a weight. A cord is wound about c and, passing over a pulley, supports a mass m.

The friction cone b is hollow and contains water, a thermometer, and a stirrer. A revolution counter is actuated by the vertical axis.

Method and Manipulation.—Fill the cup b with a known mass of water, and let its temperature be about the temperature of the room. Do not fill the cup above the tapered portion. It is

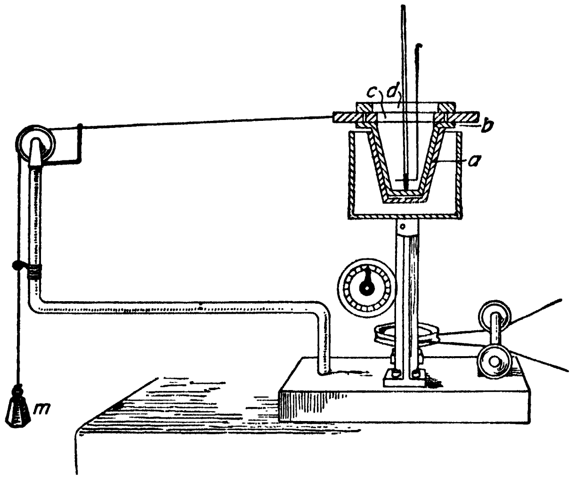


Fig. 15.

desirable to spend some time in so adjusting the friction that a fair-sized mass can be supported by it, since the best results are obtained when the rise in temperature is rapid.

This is secured by using the proper amount of oil between the cones. Record the reading of the revolution counter, and obtain a series of temperature readings at 1-minute intervals extending from several minutes before the rotation begins to several minutes after it stops. The water must be stirred constantly while the readings are being taken. Record again the reading of the revolution counter. Determine the corrected initial and final temperatures by the graphical method explained in connection with Exp. 7.

If M = the mass of a, b, and stirrer,
M_1 = the mass of the water,
m = the mass supported by the cord,
s = the specific heat of the metal in a, b, and stirrer,

n = the number of revolutions,
t = the initial temperature of the water,
t' = the final temperature of the water,
e = the water equivalent of the thermometer,
r = the radius of the wheel c,

then the work done against friction is

$$2\pi rnmg \text{ ergs,}$$

and the number of calories into which this work has been transformed is

$$(M_1 + Ms + e)(t' - t).$$

Therefore the mechanical equivalent

$$J = \frac{2\pi rnmg}{(M_1 + Ms + e)(t' - t)}.$$

CHAPTER IV

SOUND

EXPERIMENT 1

SPEED OF A TRANSVERSE WAVE ALONG A CORD

Apparatus.—A cord about 20 m long, and of about 4 mm diameter, has one end attached to a rigid support. At the other end the cord passes over a pulley and supports a mass which may

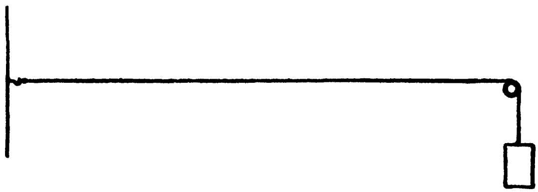


FIG. 1.

be varied from 1 to 4 kg. The pulley should have ball bearings so that the friction may be as small as possible.

Method and Manipulation.—Attach a mass of approximately 1 kg to the end of the cord. Start the wave by giving the cord a quick transverse blow near one end, and at the same instant start the stop watch. Count the number of returns of the wave which are clearly perceptible, and stop the watch at the last return counted. The number of returns times twice the distance between the supports divided by the time in seconds gives the speed. The distance must be measured along the curve of the cord for each load. Use a steel tape.

The speed of the wave is $v = \sqrt{f/m}$, where f is the tension of the cord in dynes and m the mass per unit length of the cord. Determine the mass m by weighing and measuring the entire length of the cord. For f the mass supported by the cord multiplied by the local acceleration due to gravity may be used. From these values compute v, the speed. This should check with

the observed value obtained, as directed above. Repeat by using a mass of approximately 4 kg.

EXPERIMENT 2[1]

TO TEST EXPERIMENTALLY THE EQUATION $n = 1/2S\sqrt{f/m}$

Apparatus.—A wire which is attached to the table at one end passes over two supports and carries a mass at the other end. At the middle point of the wire is a mechanism for determining the number of vibrations. As the wire vibrates, it makes and breaks the current through an impulse counter which registers the total number of vibrations.

Method and Manipulation.—1. Determine m, the mass of the wire per unit length; S, the length of the wire between the two supports; and f, the tension in the wire in dynes. Substitute these values in the equation, and obtain n, the number of vibrations of the wire per second.

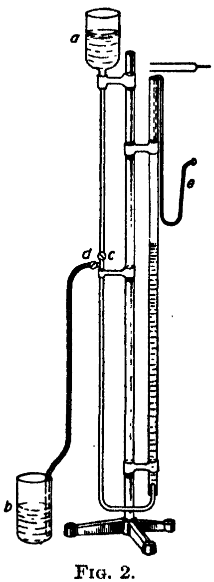


FIG. 2.

2. Cause the wire to vibrate, and observe the number of vibrations as registered by the impulse counter in a given time. From this determine n, the number of vibrations made by the wire in a second, and compare this value with that obtained from the equation.

3. Repeat, using a different tension.

EXPERIMENT 3

SPEED OF SOUND IN AIR

Apparatus.—A large vertical glass tube has at its upper end an ear tube e and a tuning fork, as shown in Fig. 2. The lower end is connected to the vessels a and b by means of tubes, as shown in figure. c and d are stop cocks. When d is closed and c is open, the water in the resonance tube rises. When c is closed and d is open, the water surface is lowered.

Method and Manipulation.—Allow the water to enter the tube. As it rises, the increased intensity of the sound is detected through

[1] POYNTING AND THOMPSON, "Sound," p. 82.

the ear tube. Place a rubber band at the position of the water surface when the intensity is a maximum. Repeat several times by turning the stop cocks c and d. In this way locate the position of the water surface for each point where the reenforcement of the sound is a maximum. The distance between two adjacent points of increased intensity is equal to a half wave length of the sound. Place bands at the highest and lowest positions noted, and measure the distance d between them. Repeat several times.

Let λ be the wave length, and i the number of internodes, *i.e.*, spaces between nodes, in the distance d; then

$$\lambda = \frac{2d}{i}.$$

At the open end the antinode is $0.6r$ beyond the end of the tube where r is the radius of the tube.

The speed v_1 of the sound in the tube at the temperature $t°$ of the room $= \lambda n$ where n is the frequency of the fork. The speed at 0°C is

$$v_0 = \frac{v_t}{\sqrt{1 + \alpha t}},$$

where α = the coefficient of volume expansion of a gas.

The speed v_a in free air is greater than in the tube and is obtained from the equation

$$v_a = v_0\left(1 + \frac{0.007989}{2r\sqrt{\pi n}}\right).^*$$

where r is the radius of the tube and n the frequency.

EXPERIMENT 4

SPEED OF SOUND IN A ROD. KUNDT'S METHOD

Apparatus.—The solid in which the speed is to be determined must be in the form of a long rod or tube. This is firmly clamped at its central point. A cork disk is firmly attached to one end of the rod and is placed loosely in the end of a horizontal glass tube. The other end of the glass tube contains a movable piston.

* *Phil. Mag.*, September, 1894, p. 254.

Method and Manipulation.—Distribute cork dust inside the tube so as to form a thin layer throughout its length. Stroke the free half of the rod with a resined cloth, thus producing in it longitudinal vibrations. The cork disk transmits these vibrations to the air in the tube. By means of the piston at the other end, the length of the vibrating air column is adjusted until stationary waves are formed. Since only compression and rarefaction take place at the nodes, while at the internodes the air sweeps to and fro, the dust at the former remains undisturbed. This affords a means of locating the nodes and determining the wave length in air of the sound emitted by the rod.

When a rod is clamped at its middle the wave length, in its own substance, of the sound it emits is twice the length of the rod.

Measure the distance d between the extreme nodes in the glass tube.

Let i = number of internodes in length d,
S = length of rod,
v_t = speed in air at temperature of room,
v_s = speed in solid,
λ_a = wave length in air,
λ_s = wave length in solid.

Since the periods of the waves in the rod and in the air are the same,

$$v_s = \frac{\lambda_s}{\lambda_a} v_t,$$

where

$$\lambda_a = \frac{2d}{i},$$

and

$$\lambda_s = 2S.$$

Use 332 m/sec as the speed in air at 0°, and from this determine v_t as in Exp. 3.

Make several determinations of λ_a.

Substitute the values obtained, and solve for v_s.

The difference between the speed in free air and in a tube is disregarded in the foregoing.

EXPERIMENT 5

FREQUENCY OF A FORK BY MEANS OF STATIONARY WAVES PRODUCED BY IT IN A CORD

Apparatus.—A braided-silk line of about 130 cm length and ½ mm diameter is attached to an electric fork and supports a weight pan, as shown in Fig. 3.

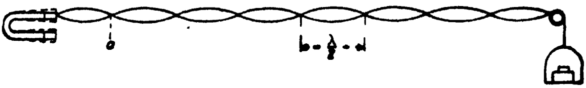


FIG. 3.

Manipulation.—Adjust the masses until the number of loops is large. Measure the distance from pulley to the first node *a* from the fork. From this length obtain the value of the wave length λ. Obtain also the mass per unit length by weighing accurately a longer length of the same line. The tension f is the mass supported by the line times the local value of the acceleration due to gravity. Compute the frequency n from the relation

$$n = \frac{1}{\lambda}\sqrt{\frac{f}{m}}.$$

Then increase the mass until the number of loops is one less, and again determine n. Continue thus until the number of loops becomes as small as possible. From these results determine the average value of n.

EXPERIMENT 6

FREQUENCY OF A TUNING FORK BY THE GRAPHICAL METHOD

Apparatus.—The tuning fork is adjusted so that it will trace its vibrations by means of a stylus on the smoked surface of a rotating drum. A timing stylus giving seconds is adjusted so that its tracing is parallel and close to the tracing of the fork. The timing stylus is actuated by means of an electromagnet through which a current is passed when the pendulum of a clock or a synchronous motor provided with a reduction gear closes the circuit.

Manipulation.—Cover the surface of the drum with glazed paper, and smoke the surface by holding a luminous flame against

it while the drum is rotating. Instead of the ordinary glazed paper, a specially prepared paper sensitive to the contact of a metal point may be used. A fine metal stylus is then attached to the fork and similarly to the timing lever.

Let the stylus of the fork and timing stylus make their tracings on the surface as directed above.

Determine the frequency by counting the total number of vibrations made by the fork during the total number of seconds recorded by the timing stylus.

Also, determine the number of vibrations during each single interval recorded by the marker. If the mercury surface in the

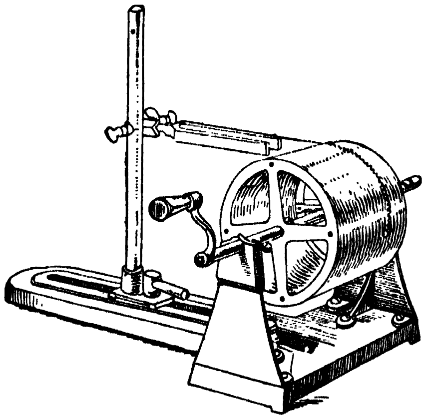


FIG. 4.

standard clock is not exactly in the center of the swing of the pendulum, the intervals will be of two different lengths, occurring alternately. Therefore find the average number of vibrations for each series of alternate intervals, and take the mean of the two averages for the final result.

CHAPTER V

LIGHT

EXPERIMENT 1

PHOTOMETRY

To investigate the illuminating power of various incandescent electric lamps by means of a bunsen photometer.

Apparatus.—A bunsen photometer designed for the use of an electric lamp as a standard, and with a socket for the lamp to be investigated which is capable of rotation about an axis at right angles to the axis of symmetry of the lamp. The standard lamp

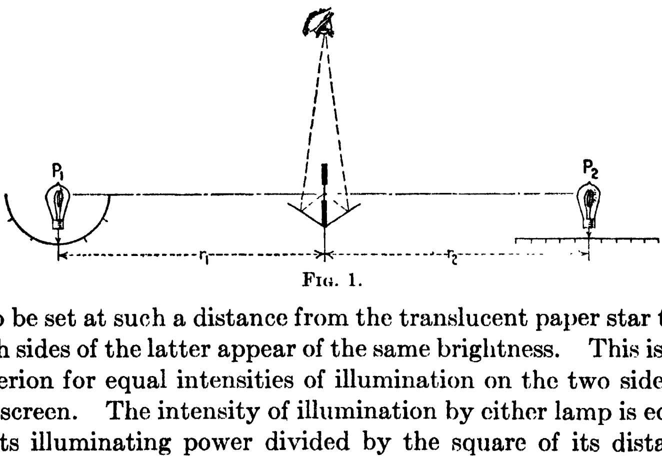


FIG. 1.

is to be set at such a distance from the translucent paper star that both sides of the latter appear of the same brightness. This is the criterion for equal intensities of illumination on the two sides of the screen. The intensity of illumination by either lamp is equal to its illuminating power divided by the square of its distance from the star. If P_1 and P_2 are the respective powers, and r_1 and r_2 are the distances, then for equal intensities at the star

$$\frac{P_1}{r_1^2} = \frac{P_2}{r_2^2}.$$

This gives a convenient method for comparing the illuminating powers of two sources, one of which is known. The unit is the candle power.

Method and Manipulation.—Owing to retinal fatigue of the eye, the adjustment will vary according to which side of the star is originally the brighter. Hence settings should always be made in pairs, first with approaching and then with receding standard. The average reading may be used in the computations.

Adjust the voltage drop across the lamp terminals to 110 volts by means of the rheostat in each circuit. Make settings for computing the lateral candle power of several lamps of different rating. The lateral candle power is that on a line at right angles to the axis of the lamp.

Make settings for computing the candle power of any one lamp in different directions in a plane containing the axis of the lamp. Steps of 15 deg are advised. If the setting is impossible for any of the chosen directions on account of the limited range of motion of the standard, proceed as follows: Place a plate of translucent glass on the side of the standard. The normal transmission coefficient of this sheet must be determined by measuring the candle power of any lamp in the same direction with and without the insertion of the glass on the side of the standard. The ratio of the true candle power to the value when the plate is inserted is the coefficient required. This coefficient should be determined for several different degrees of illuminating power and used in correcting the values where it has to be used.

Calculations.—Find the values of the candle power of the unknown lamp in the various directions described, assuming that the candle power of the standard is known (this will be given). Plot in polar coordinates the distribution curve for the lamp whose candle power in various directions has been computed. Designate each lamp by its rating and number. If a translucent sheet of glass had to be used, give its normal transmission coefficient (*i.e.*, the fraction of the light incident normally, which is transmitted in the same direction).

EXPERIMENT 2

SPHERICAL MIRRORS

Apparatus.—The spherical mirror *M* is mounted so that it may be rotated about a vertical axis, also so that it may be moved along the metric scale that supports it. A vertical rod *o* serves

as the object. Back of the rod o and mounted on the same slide is a plane mirror N placed vertically and at an angle of 45 deg with the supporting scale. The purpose of this mirror is to produce a bright background against which the image may be clearly seen. The image I is located by adjusting a vertical rod c until

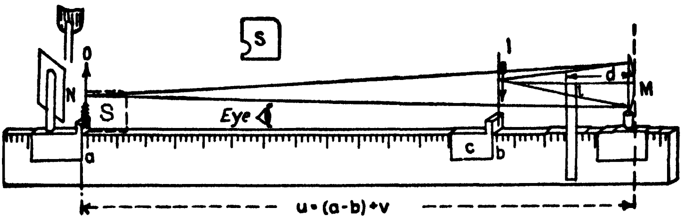


Fig. 2.

there is no parallax between it and the image. The position of the mirror is determined by placing the end of the rod l against the surface of the mirror and obtaining the reading on the scale opposite the vertical edge attached to l and adding to this reading the length d. The readings a and b for the position of the object and image on the scale are obtained by means of the square S.

A. Real Image

Method.—In the general equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \frac{2}{R} = \text{a constant},$$

u is the distance of the object from the mirror, v the distance of the image, f the focal length, and R the radius of curvature of the mirror. Determine v for each of several values of u, and substitute in succession the values of each set in the general equation. Solve for f in each case, and average. The equation is verified if the several values obtained for f are constant. Draw a graph of v as a function of u.

B. Virtual Image

Apparatus.—Same as in A, except that the concave or convex mirror has a small hole at the center or a small area from which the silvering has been removed.

Method.—To obtain the distance of the virtual image, place the rod c behind the mirror, and adjust until there is no parallax between the image and the rod as seen through the hole or unsilvered area.

Verify the equation for each of the two mirrors, as in case A.

EXPERIMENT 3

TO MEASURE THE FOCAL LENGTHS AND RADII OF CURVATURE OF SPHERICAL MIRRORS

Method.—Find the distance of the image from each of the two kinds of mirrors used in Exp. 2, when the rays coming from the object are nearly parallel. For this purpose use an object situated at a great distance. Then in each case $f = v$, and $R = 2f$.

Also, find R by moving the mirror toward the object until $u = v$, *i.e.*, until there is no parallax between the object and its image. Then $u = v = R$.

The values thus obtained for and R should check with those obtained in A and B of the previous experiment.

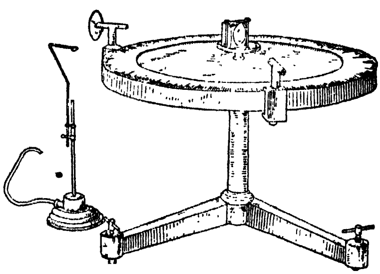


FIG. 3.

EXPERIMENT 4

REFRACTIVE INDEX OF GLASS
APPROXIMATE METHOD

Apparatus.—A spectrometer consisting of the following parts: a force table graduated in degrees or half degrees; a collimating tube with attached screen to cut off extraneous light; a prism mounted so that it may be rotated about the vertical axis of the force table; a sighting tube with peep sight. A bunsen burner with a bead of NaCl supported so as to touch the side of the flame.

Method and Manipulation.—Clamp the collimating tube so that its axis points toward the center of the force table (the point on top of the prism clamp is directly over the center), and place the sodium flame so that light passing through the collimator will strike one face of the prism. Place the sighting tube so that the image of the collimating tube formed by the prism will appear in line with the center of the disk when viewed through the sighting tube. Find by trial the position of the prism and sighting

tube for minimum deviation (the faces of the prism will then be equally inclined to the axes of the collimating and sighting tubes, respectively). When this adjustment has been correctly made, a slight rotation of the prism in *either* direction will cause the image of the collimating tube to move in a direction of greater deviation. Record the positions on the circular scale of both the collimating and the sighting tubes. Rotate the prism, and readjust for minimum deviation on the other side of the direction of the incident light, and record the new position of the sighting tube. Repeat several times for different positions of the collimating tube so that the same scale readings will not recur.

Calculations.—Compute the average angle of minimum deviation from all the readings taken. Calling this average value D and the angle at the vertex of the prism A (this is 60.0 deg, to the accuracy indicated, for all the prisms supplied), the required refractive index for the yellow lines of the sodium spectrum, which supply practically all the light used in making the settings, is given by the equation

$$\mu = \frac{\sin (A + D)/2}{\sin A/2}.$$

Compute the value of μ. For a more accurate method see Exp. 12.

EXPERIMENT 5

CONVEX LENSES

Apparatus.—Convex lenses and apparatus as shown in Fig. 4. The reading on the scale for the midway point of the thickness of the lens must be determined. This is done by placing the end of

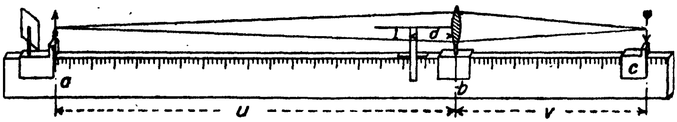


FIG. 4.

the rod l against the surface of the lens and obtaining the reading on the scale for the vertical edge, adding or subtracting the distance d and one-half the thickness of the lens as determined by placing the rod against the other surface of the lens. Adjust

the image finder c until there is no parallax between it and the image.

Determine the scale readings for the object a and for the image finder c by means of the square.

REAL IMAGE

In the general equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} = \text{a constant},$$

u is the distance of the object, v the distance of the image from the center of the lens, and f the focal length of the lens. The distances u and v in the case of a convex lens giving a real image are positive. If any one of them should be on the other side, as in one of the following experiments, it is to be taken with the negative sign.

Thus u and v are positive for a real image, and f must therefore be positive for a converging lens.

Determine v, by parallax method, for each of several values of u and solve for f and average. The settings can be made more accurately if the image finder is made to cross the image at a very acute angle.

EXPERIMENT 6

TO MEASURE THE FOCAL LENGTH OF CONVEX LENSES. ALTERNATIVE METHODS

Method.—Find by the parallax method the distance of the image of a distant object for the convex lenses used in Exp. 5. This requires the determination of the scale readings giving the positions of the image finder and the middle point of the thickness of the lens in each case. Make several settings, using different objects all more than 30 m away. This gives directly the focal lengths of the lenses, and the values obtained should check with those obtained in Exp. 5.

With a plane mirror behind the lens also find f by moving the object until there is no parallax between it and its image. The rays striking the mirror retrace their paths and hence must be parallel. The object is then at the principal focus of the lens.

EXPERIMENT 7

TO MEASURE THE FOCAL LENGTH OF CONCAVE LENSES AND LENSES OF GREAT FOCAL LENGTH

Method.—The method of Exp. 5 depends on the location of a real image on a scale 1 m in length. Concave lenses never form a real image for a real object, and even convex lenses never form a real image closer to the object than four times their focal length. This limits the applicability of the previous method to convex lenses of focal length less than 25 cm.

The focal length of a weak or diverging lens is most conveniently found by combining it with a stronger converging lens. If the lenses are both thin, it is sufficient to place them face to face and treat them as a single lens with a focal length f, which is measured. If the strongest lens has a focal length f_1, and this is known or measurable, then

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1},$$

from which f_2 may be calculated for any type of lens.

If the thicknesses of the individual lenses are such that the face-to-face combination cannot be considered the equivalent of a thin lens, then it is better to separate them by a space of several centimeters. First take the strong converging lens alone, and locate its real image at the distance v_1. The other lens to be tested is then inserted at an arbitrary distance d from the first, and the new image distance v_2 to this lens is measured. The image produced by the first lens serves as a virtual object for the second lens at a distance $u_2 = -v_1 + d$ from its center. The focal length f_2 is then found by the relation:

$$\frac{1}{f_2} = \frac{1}{-v_1 + d} + \frac{1}{v_2}.$$

This method is especially applicable to spectacle lenses.

EXPERIMENT 8

THE MAGNIFYING POWER OF OPTICAL INSTRUMENTS

From a point upon an object the eye receives a cone of rays the angle of which becomes greater as the distance between the

eye and object becomes less. The point is clearly seen when the eye is able to bring this cone of rays to a focus upon the retina. The distance at which an object is seen clearly and with the highest magnification by the unaided eye is in most cases about 25 cm. If the object is within this distance, it can still be seen distinctly by the aid of a lens which converges the rays so as to make them appear to come from a greater distance which by proper adjustment of the lens may be made that of the most distinct vision (25 cm). This lens, or simple microscope, is then said to be focused for the object.

The angle between the axes of the two cones of rays received by the eye from two extreme points of the object or image determines the apparent size of the object or image.

The magnifying power of any optical system with an accessible object is defined as the ratio of the apparent size of the final image when viewed by the eye at the most favorable distance (25 cm) to the apparent size of the object when most favorably placed for observation by the unaided eye.

In the case of microscopes of all kinds the object is considered to be most favorably placed when at 25 cm from the eye.

In the case of a single lens or mirror forming a real image and in the case of a telescope with a very distant or inaccessible object the magnifying power is defined as the ratio of the angle subtended by the image at the most favorable distance to the angle subtended by the object at its actual distance.

1. Magnifying Power of a Convex Lens When the Object Viewed Is at a Great Distance.—The image in this case is formed at practically the principal focus of the lens, and the distance of the object from the lens (D, Fig. 5) may be considered as practically the distance of the object from the eye. In observing the image I, the eye is placed 25 cm from it, *i.e.*, at the distance of most distinct vision. The angle formed at the eye by the image I is approximately equal to $\frac{I}{25}$, and the angle formed by the object directly is

$$\frac{A}{D + F + 25} \simeq \frac{A}{D} = \frac{I}{F}.$$

Therefore the magnifying power $M = \frac{\beta}{\alpha} = \frac{I}{25} \div \frac{I}{F} = \frac{F}{25}.$

Manipulation.—Place a scale horizontally at a distance of 25 cm from the eye. Adjust the lens so that there is no parallax between this scale and the image I of the distant object.

1. Determine the distance F, and compute the magnifying power M.

2. With the eye 25 cm from I, measure the distance between two points on the image by means of the scale at I. Remove the

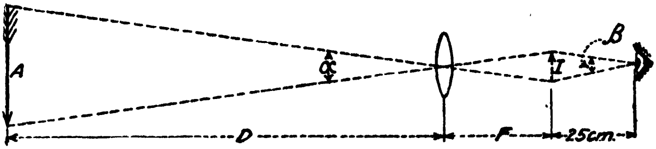


FIG. 5.

lens, and with the eye and scale in the same position 25 cm apart measure the distance between the same two points on the object directly. The ratio of these distances should check with the computed value of M.

2. Magnifying Power of a Convex Lens When the Object Viewed Is within the Principal Focus (Simple Microscope).—The image produced is a virtual image, and the microscope is said to be in focus for the object when the image is seen most distinctly, *i.e.*, when it is at the distance of most distinct vision, or about 25 cm from the eye. The object observed is nearer the eye than the image and for comparison of magnification must be assumed to be placed 25 cm from the eye or at the same distance as the image. The eye is assumed to be placed very *near the lens* so that the angle formed at the center of the lens is nearly the same as that formed at the eye. Then

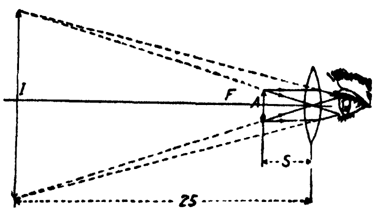


FIG. 6.

$$M = \frac{I}{25} \div \frac{A}{25} = \frac{I}{A} = \frac{25}{S}.$$

Manipulation.—Focus the lens on a vertical graduated scale A, and place another similar scale 25 cm from the lens. With one eye observe the image of the first scale, and with the other view the second scale directly. Adjust the distance S by moving A

until the image of the first scale appears beside the second scale and has no parallax with it.

Move the eyes vertically in testing this adjustment, since their accommodation for different distances permits independent rotations about vertical axes, causing apparent lateral shift.

1. Determine the value of S, and calculate the magnifying power of the lens from the equation

$$M = \frac{25}{S}.$$

2. With one eye observe the image of the first scale, and with the other eye view the second scale directly. Determine the number of scale divisions on one that corresponds to a given number on the other. The ratio of these is the magnification and should check with the computed value.

3. Magnifying Power of a Compound Microscope.—The object A is a small distance beyond the focus of the objective N, so as to form a real image I_1 at some distance from the objective. This image is within the principal focus of the eyepiece X which acts as a simple microscope.

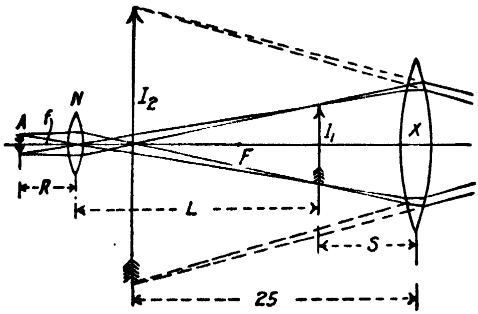


FIG. 7.

The real image is larger than the object, its magnification being

$$M_1 = \frac{I_1}{A} = \frac{L}{R}.$$

The magnification of this image by the eyepiece, as in Case 2, is

$$M_2 = \frac{25}{S}.$$

The total magnification of the object is

$$M = M_1 M_2 = \frac{25L}{RS}.$$

This assumes that the eye is very near the lens.

Manipulation.—Determine the distance S for the lens X, to be used as the eyepiece, as directed under Manipulation in Case 2 above. Having done this, leave the scale at I_1 in position, and remove the lens X. Place a vertical scale A and the lens N, which is the objective, so that a real image of A is formed. Then adjust A and N until there is no parallax between the real image of A and the scale at I_1. Replace the lens X. The final image I_2 of A is then seen at a distance of 25 cm. from X, the eye being close to the lens X.

1. Measure the distance R, L, and S, and compute the magnification M.

2. With one eye observe the image I_2 through the lens, and with the other eye observe directly a scale placed 25 cm from the eye, either on the same support as the other parts or held at one side of it. The ratio of the number of divisions on the scale at 25 cm that correspond to a given number on I_2 is the magnification and should check with the computed value of M.

4. Magnifying Power of an Astronomical Telescope.—The object A (Fig. 8) is assumed to be at a considerable distance from

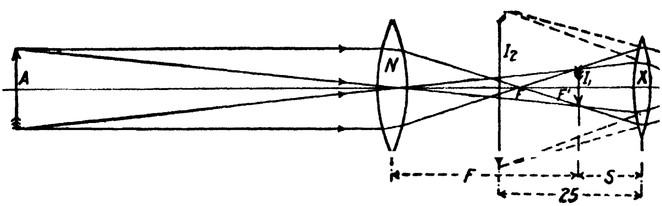


Fig. 8.

the lens N. The magnifying power of the objective, as shown in Case 1, is $M_1 = \frac{F}{25}$. When the image falls within the principal focus of the eyepiece X, the magnification of this image by the eyepiece is $M_2 = 25/S$, as shown in Case 2.

The total magnifying power $M = M_1 M_2 = F/S$.

Manipulation.—The procedure is the same as under Manipulation in Case 3, except that the lenses X and N are interchanged, and a distant object is used instead of the scale at A.

1. Determine the distances F and S, and compute the magnification M.

2. Hold a scale at a distance of 25 cm from the eyepiece. On this scale measure the distance between two points on the image by viewing the image with one eye and the scale with the other. Then with the eye and scale in the same position measure on the scale directly the distance between the same two points on the object. The ratio should check with the computed value for M.

5. Terrestrial Telescope.—Place the reversing lenses between the objective and the eye lens and at a distance from the real image equal to the focal length of the first reversing lens. Since

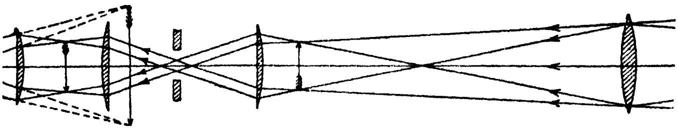


FIG. 9.

the two lenses are alike, the second image will be formed at a distance from the second lens equal to its focal length. Place the eye lens at a distance S from the second image. Observe the erect image.

The magnifying power is not affected by this use of the reversing lenses.

EXPERIMENT 9

ADJUSTMENT OF THE SPECTROMETER

The spectrometer consists of a circular graduated disk mounted horizontally on a vertical axis supported by a leveling base. Two arms extend horizontally from the axis. One supports a telescope, and the other a collimator. The arm supporting the telescope may be unclamped from the vertical axis and rotated so as to make any desired angle with the collimator. Beneath the telescope and adjacent to the disk is a tangent screw for moving the telescope slowly. Surrounding the graduated disk is a rim which moves with the telescope and on which are two verniers placed 180 deg apart.

The telescope is made of three tubes. The first is the eyepiece and contains a thin glass plate set at an angle of 45 deg with the axis of the telescope and directly opposite a round opening in the side of the eyepiece. The second contains the cross hairs and is moved in the third by means of a rack and pinion. The third contains the objective and is fixed to the supporting arm. The collimator consists of two tubes. One is fixed to the arm and

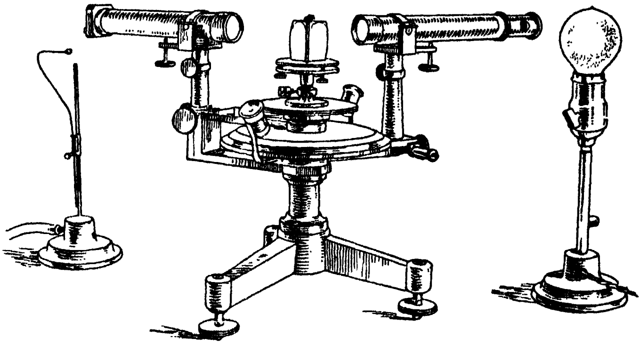


Fig. 10.

contains a lens similar to the objective of the telescope. The other slides in the first and contains an adjustable slit.

1. Focus the Eyepiece on the Crosshairs.—This is done by moving the eyepiece in or out until the cross hairs are seen distinctly. It is important for comfortable observation without eyestrain to make this adjustment very carefully.

2. Focus the Telescope for Parallel Rays.—In the experiments to follow it is necessary that rays coming from the collimator be parallel, and hence the telescope must be adjusted for parallel rays. This may be done by focusing the telescope on a distant object. This adjustment is to be perfected later by a more exact method.

3. Alignment.—Align the telescope and a collimator with the center of the spectrometer disk. This is done by unclamping the telescope or the collimator from its supporting arm, sighting along its barrel, and turning until it points to the center of the disk.

4. Level the Disk of the Spectrometer.—Place a level on the disk so that it is parallel to a line through two of the screws in the

base, and level. Then place the level at right angles to its first position, and level by turning the third screw.

5. Level the Telescope and the Collimator.—Place the level on top of the barrel of the telescope, and adjust by turning the vertical screw underneath. In a similar manner level the collimator.

6. Adjust the Telescope for Parallel Rays.—Place a piece of plate glass or a mirror that can reflect from either side at the center of the spectrometer disk, and adjust by means of the screws underneath until it appears to be perpendicular to the axis of the telescope. Place a light at the side of the eyepiece so that the rays enter the opening, as shown in Fig. 10. The glass plate in the eyepiece reflects a portion of the rays down the barrel of the telescope. In passing the cross hairs some of the rays are cut off. The remaining rays pass through the objective to the glass surface or mirror where a portion is again reflected. Since the face of the glass was placed nearly perpedicular to the telescope, a portion of the reflected rays will again enter the telescope. If the cross hairs are in the principal focus, rays proceeding from them are parallel while passing from the objective to the reflecting surface. In this case the reflected rays will also be parallel and on entering the telescope will be brought to a focus in the plane of the cross hairs, forming an image of them which may be seen through the eyepiece. Therefore, when the telescope is adjusted for parallel rays, the cross hairs and their image are in the same plane, and no parallax is observed when the eye is moved in front of the eyepiece.

7. Adjust the Telescope Perpendicular to the Axis of the Spectrometer Disk.—When the reflecting surface is perpendicular to the axis of the telescope, rays coming from the objective to the reflecting surface parallel to the axis of the telescope will be reflected back in their paths. In this case, since the intersection of the cross hairs is in the principal focus, rays coming from the intersection will retrace their paths and hence come to a focus at the intersection. The superposition of the cross hairs and their image is therefore the test for perpendicularity.

Place a piece of plate glass with polished and parallel surfaces on the stand at the center of the spectrometer disk, and adjust one face so that it is perpendicular to the telescope. Then turn

the other face toward the telescope by turning the spectrometer disk 180 deg, and also adjust for perpendicularity by leveling out half of the error by means of the telescope leveling screw and the other half by means of the leveling screws supporting the plate. Then, turning back to the first face, readjust in the same manner. This process is repeated until both faces are perpendicular to the telescope. The telescope is then perpendicular to the axis of rotation of the spectrometer disk, and neither its level nor its focus should be changed.

8. Adjust the Faces of a Prism so that They Are Perpendicular to the Axis of the Telescope.—Replace the piece of plate glass by a prism, and adjust the level of the plate on which the prism rests until both faces of the prism are perpendicular to the telescope. It is essential not to change the level of the telescope. After one face of the prism has been adjusted, and the other face is turned toward the telescope, the leveling must be carried out so as not to change more than is necessary the perpendicularity of the face previously adjusted. This is done by turning the proper screw.

Eccentricity.—If the rim that carries the verniers does not rotate about the center of the graduated disk, the zero line on the vernier will, for the same degree of rotation, move over an unequal number of divisions on the disk for different portions of the circumference and hence give rise to an error in angular measurements. This error is partly eliminated by measuring the angle with each of the two attached verniers placed 180 deg apart and taking the average.

EXPERIMENT 10

ANGLE OF INCIDENCE IS EQUAL TO THE ANGLE OF REFLECTION

Apparatus.—A spectrometer and a glass prism.

Method and Manipulation.—Adjust the spectrometer according to the directions given in Exp. 9. Place the prism as indicated in Fig. 11. Move the telescope into the position N, and adjust the face of the prism so that it is perpendicular to the axis of the telescope. Obtain the reading N for this position. Then turn the telescope into the position R so that the intersection of the cross hairs coincides with the center of the reflected slit.

Obtain the reading R for this position. Then remove the prism, and turn the telescope into line with the collimator, and obtain the reading S for this position. Rotation of the divided circle or slippage of the telescope must be carefully avoided. Then

$$i = 180 - (S - N),$$

and

$$r = R - N;$$

then if

$$i = r,$$
$$R - N = 180 - (S - N).$$

Read both verniers, keeping their readings separate, and finally take the mean of the two values so obtained. Designate

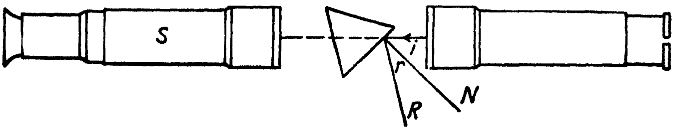


FIG. 11.

the verniers as to the right or left of the telescope, since they always have the same positions relative to it.

To obtain the actual angle of rotation of the telescope it must be noted whenever a vernier passes through zero and in what direction. If it passes through zero in the direction of increasing angles, 360 deg must be added to the second reading; otherwise it is to be subtracted.

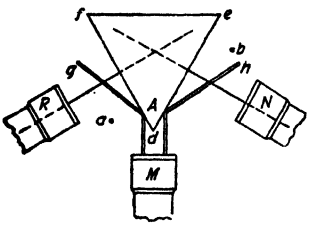


FIG. 12.

EXPERIMENT 11

ANGLE OF A PRISM

Apparatus.—A spectrometer and a prism.

Method and Manipulation.—Adjust the spectrometer. Since the angle between normals to two faces of the prism has to be measured, it is necessary that the prism be perpendicular so that the cross hairs and their reflected image will coincide for each face, and so that the prism faces are parallel to the axis of rotation.

When the prism has been made perpendicular, as directed in Exp. 9, superpose the cross hairs and their image for one of the faces, and record the reading for the normal R. Then turn the telescope to the other face, and record the reading for the normal N. The angle of the prism is

$$A = 180 - (R - N).$$

It is easier here to leave the telescope and lamp fixed, turning the prism and graduated disk, the final settings being made with the aid of the tangent screw beneath the collimator.

A second method for obtaining the angle of the prism is to place the collimator at M (Fig. 12) and measure the angle between the reflected rays g and h. This angle is twice the angle of the prism.

It is necessary here to turn the telescope, and the appropriate tangent screw is the one beneath the telescope. In either method keep the readings on the two verniers separate. A good procedure is to read the degrees of one of the verniers only, *e.g.*, the one on the right, and to use the mean of the minutes and seconds as given by both verniers. This gives the same final result for the angle measured and is less cumbersome than averaging the entire readings, as a simple trial will show.

EXPERIMENT 12

REFRACTIVE INDEX OF GLASS FOR SODIUM LIGHT

Apparatus.—A spectrometer and the glass prism whose angle was determined in Exp. 11; a bunsen burner; and a bead of NaCl on a wire. A mercury arc may be used to advantage instead of the sodium light. Make measurements using the green line which has a wave length of 5461 Ångström units.

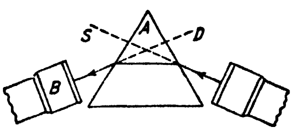


Fig. 13.

Method and Manipulation.—Adjust the spectrometer as before, adjusting the prism so that its reflecting faces are perpendicular to the disk of the spectrometer. To measure the angle of minimum deviation, turn the prism into the position indicated in Fig. 13, and allow sodium light to pass from the

collimator through the prism and into the telescope on the other side. Turn the prism and telescope until the refracted slit is seen. It will be found on turning the prism in one direction that the image of the slit moves toward the axis of the collimator produced, stops, and then returns. The turning point, therefore, marks the direction of the beam when it is deviated the least, and the angle between this and the original direction is the angle of minimum deviation to be determined. Place the intersection of the cross hairs on the image of the slit when the latter is at the turning point. Record the reading B for this position. Rotate the telescope and prism for minimum deviation on the other side of the direction of the incident light, keeping the divided circle clamped. Record the reading B' for this position. The difference between B' and B gives twice the angle of minimum deviation D, and the refractive index

$$\mu = \frac{\sin \frac{1}{2}(A + D)}{\sin \frac{1}{2}A},$$

where A is the angle of the prism.

EXPERIMENT 13

CALIBRATION OF A SPECTROSCOPE

Apparatus.—A spectrometer, bunsen flame, platinum wires, and solutions of chlorides of lithium, strontium, thallium, barium, and others. A mercury arc may be used instead of or in addition to the flame.

A spectroscope is an instrument for observing the wave length or color composition of the light emitted by various sources. It usually has fewer adjustments than a spectrometer and, instead of an angular scale, has a linear scale in the field of the telescope. A spectrometer may be used as a spectroscope in the following manner.

Method.—The spectrometer is adjusted, and the prism is set for minimum deviation for the sodium lines. The prism and divided circle are then clamped for the remainder of the experiment. The telescope is left free to rotate, and its position is determinable by the reading of one of the verniers, *e.g.*, *R*.

Before a spectrometer can be used for spectrum analysis it must be calibrated. This is done by observing a set of simple

spectra containing known wave lengths and determining the scale reading when the intersection of the cross wires is set on certain standard lines. The intersection of the wires should be set on the images of the fixed edge of the adjustable slit, or if both edges move it should be set midway between them. Convenient spectrum lines to use are the Fraunhofer lines in sunlight or the emission lines of salt vapors in a bunsen flame or in a carbon arc or electric discharges in low-pressure tubes. The simplest and most convenient source is furnished by evaporation of salts from a platinum wire in a bunsen flame. The proper method is always to use one wire for only one substance. After dipping the wire into the solution, it should be placed in the flame with end pointed downward and so that the middle heats up first and drives the solution into the tiny spiral at the end. Two students may to advantage work together and alternate in taking readings and manipulating the flame. Other methods may be used for vaporizing the solutions. Prominent flame lines which may be used for calibrating purposes, and their respective wave lengths, in Ångström units, are as follows:

Lithium	6708	Brilliant red
Sodium	5896, 5890	Orange, a double line
Thallium	5351	Yellowish green
Strontium	4607	Blue
Mercury arc	5780	Yellow
	5461	Green
	4916	Greenish blue
	4358	Deep blue
	4077	Violet
	4047	Violet

Observations.—Observe the vernier settings for these lines, and plot on cross-section paper against the wave length as ordinates. Draw a smooth curve through the points, and extend it a short distance beyond each end point. This is the calibration curve. The wave length of unknown lines may be found by observing their corresponding vernier reading and using the calibration curve. In this way find the wave lengths of the most prominent Fraunhofer lines in the spectrum of sunlight.

EXPERIMENT 14

WAVE LENGTH OF LIGHT. FIRST DIFFRACTION-GRATING METHOD

Apparatus.—A diffraction grating; a scale with a slit *a* (Fig. 14); a bunsen flame; a bead of NaCl for the production of the sodium light or a mercury arc. The apparatus is arranged as shown in Fig. 14. In this figure *a* is the slit; *G*, the grating. Images of the first three orders are shown on each side of the slit *a* but can, of course, be seen only through the grating. Figure

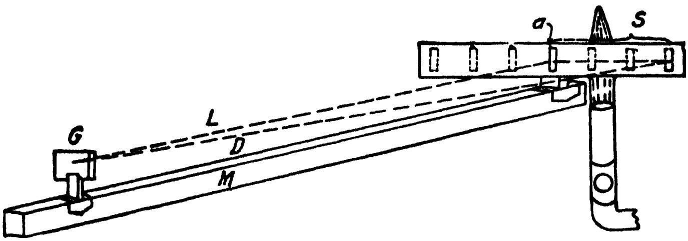


FIG. 14.

14 is a plan of the apparatus showing the notation used below. *P* is the distance from the slit to the first image, and *S* is the distance to the *n*th image. A sliding pointer is used to facilitate locating the diffraction images on the scale.

Method.—The diffracted images of the slit are observed through the diffraction grating *G* placed at a distance *L* of about a meter from the scale. Obtain the reading on the scale for the center of each image visible when the flame is freshly supplied with NaCl. Depending on the grating space, which can be computed from the number of grating spaces per millimeter (to be given in each case), one, two, three, or more images can be seen on each side of the slit by darkening the room sufficiently.

Determine the distance from the image of the highest order on one side to the image of corresponding order on the other. One-half of this distance is *S*, the distance of each image from the slit. If the grating used is large, care must be taken to have the eye near the grating and at a point directly above the scale *M*. Or, more generally, place the eye so that the line of sight passes through the center of the grating. The rays coming from the slit

can be assumed to be nearly parallel, and, if the first diffracted image is observed,

$$\lambda = d \sin \theta \text{ (Fig. 15)},$$

where d is the distance between the centers of two adjacent lines on the grating.

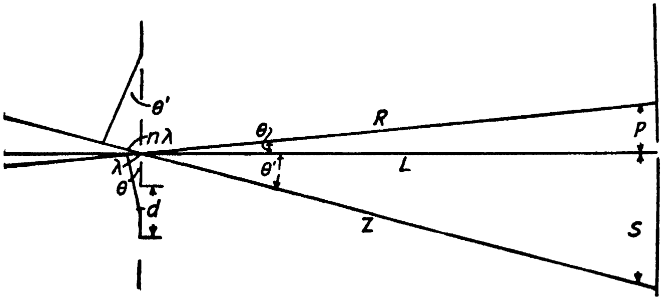


Fig. 15.

From Fig. 15 it is seen that

$$\sin \theta = \frac{P}{R};$$

$$\therefore \lambda = d\frac{P}{R}.$$

In case nth image is observed,

$$n\lambda = d \sin \theta,$$

and

$$\lambda = \frac{d}{n}\frac{S}{Z}.$$

Find the average wave length for the sodium light, using various values of L.

EXPERIMENT 15

WAVE LENGTH OF LIGHT. SECOND DIFFRACTION-GRATING METHOD

Apparatus.—A spectrometer and a diffraction grating.

Method and Manipulation.—Adjust the spectrometer as directed in Exp. 9. Then place the grating perpendicular to the collimator. This may be done in the following manner: Turn the telescope into line with the collimator, and place the cross hairs on the image of the slit; place the grating on the stand between the telescope and the collimator, and adjust by rotating and tilting

the grating until it is perpendicular to the axis of the telescope, as directed in 7 and 8 in Exp. 9. Be sure that the disk is then clamped. Then as the telescope and the collimator are in line, the grating is also perpendicular to the collimator. Allow sodium light to pass through the slit. Turn the telescope to one side until the first diffracted image of the slit enters the field of the telescope. The image is at its best when the slit and grating spaces are parallel. Place the cross hairs on the image, and record the reading N. Turn the telescope to the first diffracted image on the other side, and obtain the reading M.

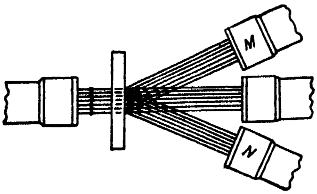


FIG. 16.

If m = the number of lines per mm on the grating,
θ = the angle of diffraction,
λ = the wave length,
d = the distance between two adjacent lines,

then

$$\theta = \frac{N - M}{2},$$

and

$$\lambda = d \sin \theta = \frac{\sin \theta}{m}.$$

If the nth image is used,

$$\lambda = \frac{\sin \theta}{nm}.$$

EXPERIMENT 16

POLARIZED LIGHT

Apparatus.—A polariscope; a crystal whose faces are not perpendicular to an optic axis (selenite or mica); a uniaxial crystal cut perpendicular to its optic axis (Iceland spar); a biaxial crystal cut perpendicular to a line bisecting the angle between the axes (niter crystal); a quartz crystal cut perpendicularly to its optic axis.

The polariscope consists of a polarizer made of a black glass plate and placed in a horizontal position. At one end of the polarizer is a vertical screen of ground glass for diffusing the light passed through it to the polarizer. From the polarizer, the light

passes to a large lens serving as a condenser near the focus of which is placed a smaller lens which renders the rays parallel. The light then enters the analyzer consisting of a Nicol prism.

Manipulation.—Assume the light waves coming from the polarizer to be vibrating in a plane perpendicular to the plane containing the incident and reflected rays.

The principal section of the Nicol prism coincides with the plane of vibration of the light that it transmits. Determine its direction in terms of the reading on the divided circle supporting the prism.

Crystal Plate in Parallel Light.

1. Turn the analyzer until its principal section is horizontal, and place a one-half wave plate of selenite or mica crystal in the

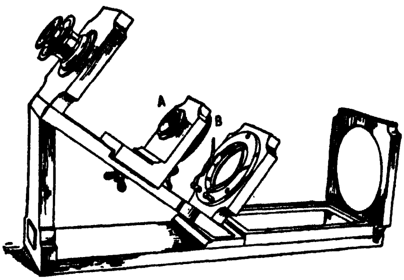


Fig. 17.

parallel rays, *i.e.*, in position A (Fig. 17). Since the crystal is not cut perpendicularly to an optic axis, it has two planes of vibration which transmit polarized white light. Rotate the crystal, and note their directions.

2. Use sodium light instead of white, and rotate the crystal until its planes of vibration make an angle of 45 deg with the vertical. Rotate the analyzer, and observe the positions for which light and darkness are obtained. Explain.

3. Replace the white light, and rotate the crystal until its planes of vibration make an angle of 45 deg with the vertical. Rotate the analyzer, and observe if two colors are obtained and if they are complementary. Explain.

4. Rotate the crystal, and observe if more than one color is obtained. Explain.

5. Remove the half wave plate, and insert in its place a quarter wave plate. Rotate the analyzer, and observe if any change takes place when sodium light is used. Explain.

6. Repeat using white light. Explain.

Uniaxial Crystals Cut Perpendicularly to the Optic Axis. Converging Light.

1. Place the principal section of the analyzer vertical, and insert the crystal of Iceland spar in the converging rays of white light, *i.e.*, position *B*. In a polariscope without lenses place the crystal just back of the analyzer. Observe the black cross. Explain.

2. Between the arms of the cross observe the colored rings. Explain.

3. Rotate the crystal, and observe if any change takes place. Explain.

4. Rotate the analyzer 90 deg. Observe the white cross, and at any point between the arms observe that the color is complementary to that seen in position 2. Explain.

Biaxial Crystal Cut Perpendicularly to the Bisector of the Angle between the Optic Axes. Converging Light.

1. Place the principal section of the analyzer vertical. Place the biaxial (niter) crystal in position *B*. Rotate the crystal until a line joining the centers of the two observed circles makes an angle of 45 deg with the vertical. Observe the hyperbola and the colored bands between its arms. Explain.

2. Rotate the analyzer 90 deg. Observe the white hyperbola, and at intervening points note that the color is complementary to that observed in position 2. Explain.

3. Rotate the crystal 45 deg., and observe the white cross. Then rotate the analyzer 90 deg., and observe the black cross. Explain.

Quartz Crystal Cut Perpendicularly to the Optic Axis.

1. Rotate the analyzer until its principal section is vertical Place the quartz crystal in position *A*, and use a sodium flame as the source of light. Rotate the analyzer until the light becomes a minimum. Observe the angle of rotation.

2. Replace the sodium flame by lights of different colors, and measure in each case the angle of rotation.

3. Use white light, and observe the different colors obtained by rotating the analyzer. Explain.

4. Place the crystal at *B*, and observe the rings. Explain.

CHAPTER VI

ELECTRICITY I

This part is designed to be the first laboratory course in electricity to be given in connection with a general course in college physics.

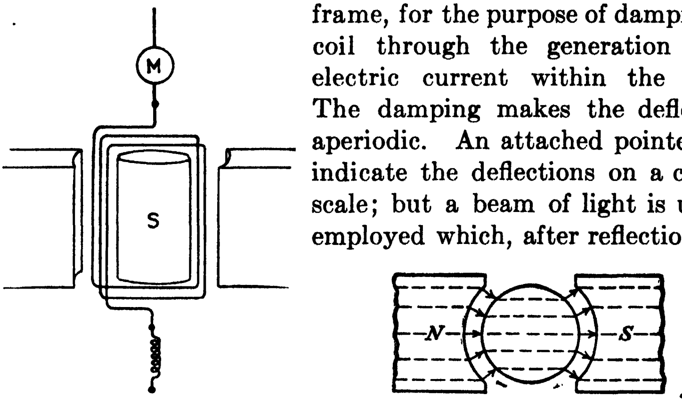
Introduction.—It is intended that the 18 quantitative experiments be reported in the usual standard form in which the results are expressed with their probable error. The first seven experiments must ordinarily be performed before they have been considered in the text. Most of the remaining 11 should not be performed until the student has mastered the theory bearing on the subject. The following schedule is suggested:

Period	Experiments	Period	Experiments
1	1	6	11, 12
2	2, 3, 4	7	13, 14
3	5, 6, 7	8	15, 16, 17
4	8, 9	9	18
5	10	10	19, 20

The qualitative experiments *A* to *T* are intended to be performed as extras during the regular laboratory periods, each at the time that the theory of the experiment is being considered in the text part of the subject. It is not intended for the student to make a formal report on them, because they are given mainly for the purpose of acquainting him more intimately with the phenomena that he has observed already in the formal lectures. The list given is suggestive only and may be varied or increased as circumstances permit. Since very little time can be spared for these experiments, the instruments must be adjusted and ready for use.

The following instruments, etc., used often, are described here for reference:

1. Moving-coil Galvanometer.—The moving-coil galvanometer consists of a coil of insulated copper wire suspended between the poles of a horseshoe magnet, as shown in Fig. 1(*a*). The upper (strip) suspension and the lower spiral suspension are of gold or phosphor-bronze and also serve as leads for the current. The purpose of the soft-iron cylinder S is to produce a uniform, radial magnetic field between the cylinder and the pole pieces, as shown in Fig. 1(*b*). The coil is usually wound on a copper or aluminum frame, for the purpose of damping the coil through the generation of an electric current within the frame. The damping makes the deflections aperiodic. An attached pointer may indicate the deflections on a circular scale; but a beam of light is usually employed which, after reflection from



(*a*) (*b*)

FIG. 1.—(*a*) Open coil of a moving-coil galvanometer; (*b*) radial field produced in the air gaps between the poles of the magnet and the soft iron cylinder in a moving-coil galvanometer.

the attached mirror M, impinges on a circular scale and gives readings proportional to the angle of deflection. This beam of light is equivalent to a weightless pointer whose length is twice the distance between the mirror and the scale. A current in the coil produces a torque which, because of the uniform radial field, is proportional in all positions to the current. This torque turns the coil until the twisted suspensions exert an equal torque in the reverse direction. The resulting deflection is as nearly proportional to the current as it is possible to produce a perfectly uniform radial field, to suspend the coil centrally, and to center a perfectly ruled scale. These conditions are fairly realized in fine instruments.

A *portable galvanometer* usually has the coil held in place by pivots in jewel bearings and controlled by spiral springs [Fig. 2(*b*)] which serve also as leads for the current. The portable galvanom-

eter is less sensitive but is more robust and convenient and therefore is used wherever a greater sensitivity is not required.

A *ballistic galvanometer* is one used to measure quantities of electricity discharged instantaneously through its coil. The coil is given energy of rotation and turns a distance proportional to the quantity of electricity discharged. The readings are called *throws* to distinguish them from continuous deflections produced by a steady current.

2. Ammeter.—An ammeter is an instrument for measuring the strength of an electric current. The direct-current ammeter

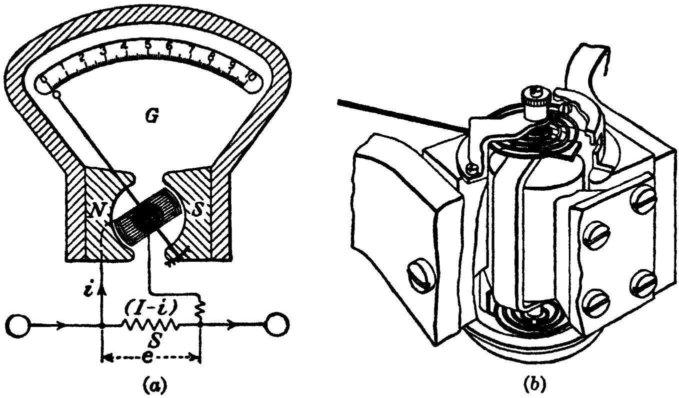


FIG. 2.—Ammeter: (*a*) diagram showing shunt *S*; (*b*) control spiral springs and coil.

is a portable galvanometer whose coil is shunted [Fig. 2(*a*)] so that only a small part *i* of the total current energizes the coil, but this part is always proportional to the total current *I* in the circuit. The scale then can be constructed to indicate the magnitude of the current in the circuit or in the branch into which the ammeter is connected. Because of the low resistance of the shunt, the ammeter has a low resistance. Therefore the introduction of an ammeter into a circuit alters but little the current in that circuit. A *milliammeter* is an ammeter whose scale is graduated in milliamperes.

The alternating-current ammeter in its various forms is described in the text. The dynamometer type consists of two coils in series, one of which can turn on its axis against the torsion of two spiral springs. The electromagnetic reaction between the

coils causes the movable coil to turn always in the same direction, because the current reverses in both coils at the same time.

3. Voltmeter.—A voltmeter is an instrument graduated to read the electric potential differences impressed across its binding posts. The direct-current voltmeter differs from the direct-current ammeter in that in place of shunting the galvanometer coil, a high resistance x (Fig. 3) is connected in series with the coil. The high resistance is for the purpose of limiting the current through the instrument so that when the voltmeter spans any section of a circuit it diverts only a small fraction of the current. The attaching of a voltmeter therefore may not diminish the

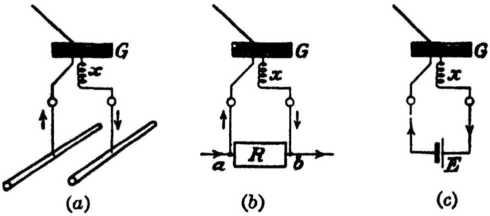


FIG. 3.—Voltmeter connected: (*a*) to measure the potential difference of mains; (*b*) to measure the potential difference between points *a* and *b*; (*c*) to measure electromotive force of a cell.

potential difference across a spanned section appreciably. The voltmeter, however, in any case, always reads the potential difference across its binding posts and therefore the potential difference across the spanned section as it is after the voltmeter is connected.

Figure 3(*a*) shows the voltmeter spanning the supply mains, and Fig. 3(*b*), the resistance R which is part of an electric circuit. When the e.m.f. of a cell is to be measured, however, the whole current flows through the coil of the voltmeter [Fig. 3(*c*)]. Since the resistance of the voltmeter is practically the whole resistance of the circuit, the potential difference across its binding posts is practically the total potential difference and therefore the e.m.f. impressed on the circuit by the cell.

An alternating-current voltmeter is a sensitive alternating-current ammeter with a high resistance in series.

4. Wattmeter.—A wattmeter measures the power consumed in any desired part of a circuit. It consists of two coils. The stationary, low-resistance field coil is connected into the circuit;

and the movable high-resistance potential coil is connected across the two points between which the power consumed is to be measured. The deflection of the movable coil then is proportional to the product of the current and the in-phase component of the potential difference and, therefore, to the power consumed in the spanned portion in either direct-current or alternating-current circuits.

5. Parallax.—When an instrument is provided with a reading mirror, the reading is taken with the eyes in a position such that the pointer appears to be directly above its own image. In instruments not provided with such mirrors, the readings are taken from a position where the line of sight passing through the pointer is perpendicular to the plane of the scale.

In adjusting a reading telescope for parallax, the eyepiece is first focused on the index line (cross hair), and then the telescope is focused on the image of the scale. After the final adjustment, both index line and scale are seen most distinctly, and there is no change in reading when one moves his head from side to side.

6. Rheostat and Slide Wire.—A rheostat usually consists of a solenoid of bare, high-resistance wire wound on an insulating cylinder. A sliding contact introduces more or less of the resistance into the circuit, and, thereby, the current can be varied as desired. Rheostats often give trouble due to poor contacts, either where the sliding-contact spring touches the solenoid or where the slide that holds the spring touches the supporting rod. Such poor contacts can often be improved by moving the contact brushes back and forth over the solenoid, by polishing the surface of the solenoid with emery cloth, or by tightening the slide carrying the contact spring. Rheostats may act as magnets whose magnetic fields are superposed on those of the electric instruments. The rheostats, therefore, should be placed as far from the instruments as is practical, preferably with their external fields at right angles to those of the instruments. If turning an energized rheostat through 180 deg does not change the readings, the disturbing influence of the rheostat is negligible. The rated current-carrying capacity should not be exceeded.

A slide-wire resistance is a rheostat in which the variable resistance is a straight wire. It is employed where small but accurate adjustments of the current are required.

7. Resistance Box.—Resistance boxes are boxes that contain a number of noninductive, known resistances of manganin wire and are used for introducing resistances of any desired known magnitude into an electric circuit. In a dial-type box, resistances are introduced into a circuit by simply turning a dial. In a plug-type box, the plugs, inserted between lugs, normally short-circuit the resistances. The removal of a plug, therefore, introduces a known resistance into the circuit. The plugs must be kept clean and when removed from the box should be placed on a clean piece of paper. They are inserted with a clockwise motion and only very slight pressure. Good contact exists when one feels a slight resistance to the turning. A test for good contacts in a plug-type box, when it is in an energized circuit, is to pass one's hand lightly over the tops of the inserted plugs; no change in the deflection of the reading instrument in the circuit indicates good plug contacts. Each coil is constructed so that it can dissipate 0.25 watts safely. This rated carrying capacity of 0.25 watts must not be exceeded, because the overheating of a coil may change its resistance permanently. It follows from

$$P = RI^2 = 0.25$$

that the maximum current permitted in any one coil is

$$I = \sqrt{\frac{0.25}{R}} \text{ amp.}$$

A 10-ohm coil therefore can carry 0.158 amp; a 20-ohm coil, 0.111 amp; and a 100-ohm coil, 0.050 amp.

The temperature coefficient of manganin resistance coils is +0.00001 and, therefore, for most purposes is negligible.

8. Battery.—Some type of storage cell should be used. Each Edison storage cell has an e.m.f. of 1.4 volts; each lead storage cell, 2.0 volts. The Edison cell has a higher resistance but is not injured so easily as the lead cell and may be left discharged indefinitely without injury. The carrying capacity of the B-4 type Edison cell is 15 amp.

9. Switch, Key, and Commutator.—A single-pole single-throw (s.p.s.t.) switch just opens or closes a circuit, while a double-pole double-throw (d.p.d.t.) switch can open or close a circuit at two points at the same time and, by reversing the bars, introduce

other elements into the circuit. It serves various other useful purposes, and one side of it may conveniently be used as a s.p.s.t. or a s.p.d.t. switch.

A single-contact key is normally open and closes or opens a circuit only when one is holding the key in its depressed position.

A commutator is a device for conveniently changing the direction of the current in any part of an electric circuit. It consists of a d.p.d.t. switch with the alternate corner lugs connected electrically. Figure 4 shows a commutator with the switch bars turned to the right. The full-line arrows show the direction of the electron flow then in the circuit. The broken-line arrows show the reversed direction of the flow in the galvanometer section when the switch bars are turned to the left.

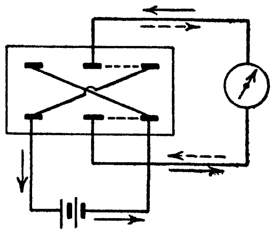


FIG. 4.

10. Binding Posts.—Binding posts are devices for conveniently attaching wires. The American type of binding post is

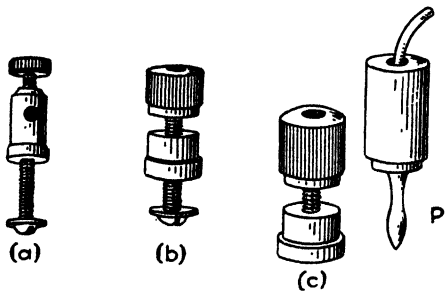


FIG. 5.

represented in Fig. 5(*a*), and the English type in Fig. 5(*b*). Another type has a provision for the insertion of plug-in prongs with which the connecting wires then must be provided. Figure 5(*c*) is a post of this type. The plug *P* is inserted into the opening shown on top.

In using the English type, the end of the connecting wire is bent into a loop so as to make a better contact when clamped. If it is necessary to insert two wires at one point, the binding post is usually provided with an extra screw or an extra nut. If it is

necessary to insert two wires in the English type of post when it has provision for only one, the ends of the wires are inserted so that they loop in opposite directions.

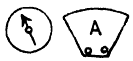
11. Connecting Wires.—A desirable connecting wire is a No. 18 or 20 rubber-covered stranded copper wire. The insulation is removed for a distance of 2 or 3 cm at the ends, and the exposed strands are twisted together to form a more compact conductor. These ends may be tinned or provided with special tips, forks, or prongs.

12. Direction of Electron Flow.—The arrows in the figures of this text show the direction of electron flow and not the conventional direction of the current.

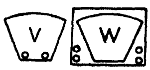
13. Instrument Numbers.—The laboratory numbers of the instruments which give the data from which the results of an experiment are calculated should be recorded for identification and included in the formal reports.

14. Symbols.—The following symbols are used in diagrams:

Battery of two cells.



Galvanometer; ammeter.



Voltmeter; wattmeter.


Noninductive resistance.


Inductive resistance.


Low-resistance rheostat.

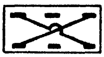

High-resistance rheostat.


Slide-wire resistance.

Known resistance or resistance box.

S.p.s.t. switch.

D.p.d.t. switch.



Commutator.

Tap key. (single-contact key).

Capacitance.

EXPERIMENT 1

AMMETER MADE FROM A MOVING-COIL GALVANOMETER

Apparatus.—Portable galvanometer; milliammeter; manganin shunt; dial resistance box; a high- and a low-resistance rheostat; slide-wire resistance; battery of two (Edison) storage cells; s.p.s.t. switch; connecting wires.

Method and Manipulation.—The galvanometer G and the resistance box R in series with it are connected across the shunt S, as shown in Fig. 6. These three instruments together, after the proper resistance has been introduced into R, constitute the constructed galvanometer ammeter. The binding posts of the ammeter are those at the extremities of the shunt. The object of the three rheostats r is to enable the current to be varied by larger or smaller steps as desired. The currents in the galvanometer and shunt together are equal to the current in the main circuit which the galvanometer ammeter is designed to measure.

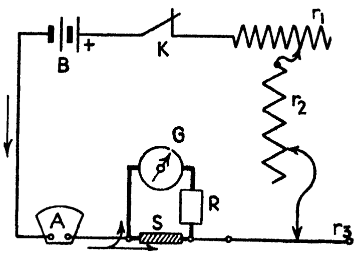


Fig. 6.

This same current energizes the standardizing milliammeter A whose construction is essentially that of the galvanometer ammeter, except that the shunt and the resistance are enclosed within the casing of the instrument.

As a precaution against possible injury to the instruments by too large a current, the rheostats r are adjusted to introduce their full resistance into the circuit, and a resistance of several hundred ohms is inserted into the box R before the circuit is closed by the switch K. The ammeter and galvanometer null readings should be zero; if they are not, the instructor should be notified. If either instrument must be used with a displaced zero, each of its readings must be corrected for this displacement. In some instruments the reading, because of friction in the bearings, depends somewhat on the direction from which the coil comes to its resting position. Errors due to such friction may be reduced by lightly tapping the instrument after each change of deflection, before taking the reading, and by approaching the deflected position always from the larger to the smaller reading.

It is desirable to make each division on the galvanometer represent a simple fractional part or multiple of an ampere or milliampere. Adjust the rheostats r and the resistance R until this condition is fulfilled, preferably when both milliammeter and galvanometer ammeter give full-scale readings. Then if, for example, 150 ma gives a deflection of 30 divisions on the galvanometer, each such division represents 5 ma. The galvanometer deflections should be from left to right. The galvanometer with the shunt and the resistance R then is an ammeter whose scale could be renumbered to read amperes or milliamperes.

Since the galvanometer deflections usually are not exactly proportional to the current, a *correction curve* can be drawn by means of which the proper corrections for any disproportionality can be made. The magnitude of these corrections is determined by comparing the galvanometer-ammeter readings by regular steps, from the largest to the smallest, with those of the standard whose readings are either assumed to be correct or are properly corrected. In place of making the corrections in terms of current, it is usually more convenient to give the disproportionality correction in terms of scale divisions. For example, if each scale division is supposed to represent 5 ma, and a current of 75.0 ma

gives a deflection of 15.2 when it should give 15.0 divisions, the correction at the point 15.2 on the scale is −0.2 divisions.

The observations and corrections should be tabulated and the correction curve drawn as in the following illustrative case:

DATA FOR CORRECTION CURVE OF GALVANOMETER AMMETER

Galv. No.______ Shunt No.______ $R = 61.0 \pm 1$ ohms
Both null points = 0.00 ± 5
Adjusted to give 1 division per 5 ma (left to right)

Milliamperes	Galvanometer ammeter		
	Should read	Observed readings	Correction
[150.0 ± 1]	[30.0 ± 1]	[30.0 ± 1]	[0.0]
125.0	25.0	24.9	+0.1
100.0	20.0	20.0	0.0
75.0	15.0	15.2	−0.2
50.0	10.0	10.1	−0.1
25.0	5.0	5.0	0.0
0.0	0.0	0.0	0.0
150.0	30.0	30.0	0.0

The scale divisions are plotted as abscissas and the corrections as ordinates, the + corrections always being placed above the

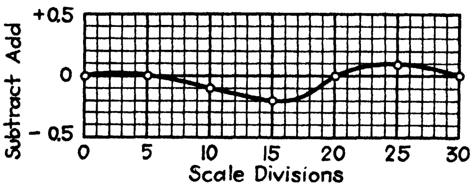


FIG. 7.

axis of abscissas, and the − corrections, below, as shown in Fig. 7. Each correction of 0.1 of the smallest divisions on the scale of the instrument to which the correction curve applies is represented by one whole small division on the coordinate paper.

EXPERIMENT 2

VOLTMETER MADE FROM A MOVING-COIL GALVANOMETER

Apparatus.—The apparatus employed in Exp. 1, except that the standard milliammeter is replaced by a standard voltmeter,

and the slide-wire resistance is discarded. The parts, however, are differently assembled.

Method and Manipulation.—It should be observed (Fig. 8) that the galvanometer and the series resistance R together form the constructed galvanometer voltmeter which is connected to the binding posts of the standardizing voltmeter V. The two instruments then have the two binding posts in common and therefore always have the same potential difference impressed upon them. This common potential difference is supposed to be measured by both instruments and is impressed on them from the main circuit M through the contact points CD. The contact C can be moved over the rheostat r_1 to obtain any desired potential difference less than the e.m.f. of the battery. To obtain the desired value of the e.m.f. more accurately, the current in the main circuit is varied by adjusting the low-resistance rheostat r_2. Before the main circuit is closed, however, a resistance of about 10,000 ohms should be placed in R to guard against injury to the galvanometer.

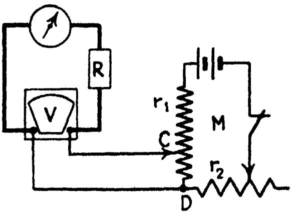


FIG. 8.

In case the galvanometer scale has 30 divisions and it is desired, for example, to make it into a galvanometer voltmeter with a 3-volt range, when the e.m.f. of the battery is only 2.8 volts, the resistance R of the galvanometer voltmeter and the common potential difference across both voltmeters are adjusted until a potential difference of 2.50 volts, recorded by the standardizing voltmeter, produces a deflection of 25.0 divisions on the galvanometer. The resistance then in R transforms the galvanometer into a voltmeter whose range is 3 volts and on which each small division represents a potential difference of 0.1 volts.

This galvanometer voltmeter is a better instrument for many purposes than a commercial voltmeter, because in the latter the resistance may be as low as 100 ohms per volt, while in a galvanometer voltmeter it is rarely less than 2000 ohms (Intr. 3).

The proportionality errors are determined and tabulated, and the correction curve drawn as in Exp. 1.

EXPERIMENT 3

RESISTANCE. AMMETER-VOLTMETER METHOD

Apparatus.—Milliammeter; galvanometer voltmeter constructed in Exp. 2; an unknown low resistance X of 5 to 10 ohms; accessories as shown in Fig. 9. The galvanometer voltmeter is now represented as the voltmeter V.

Method and Manipulation.—The potential difference across the unknown resistance X is measured for each of three different large values of the current in the circuit. From Ohm's law the unknown resistance

$$X = \frac{E}{I}.$$

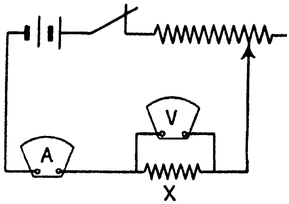


Fig. 9.

Tabulate the three sets of observations, giving the three determined values of the unknown resistance and their average.

In this experiment, and always, care must be taken not to close the circuit before placing sufficient resistance into the circuit to protect the battery and into those branches which contain instruments subject to injury by large currents.

Question.—Why must the resistance X be low?

EXPERIMENT 4

RESISTANCE. SUBSTITUTION METHOD

Apparatus.—A milliammeter (150 ma) or a portable galvanometer of low resistance; resistance box (see Intr. 7) adjustable to 0.1 ohm; a coil whose resistance is to be measured; a d.p.d.t. switch; slide-wire resistance; etc.

Method and Manipulation.—The coil X, whose resistance is to be determined, and the resistance box R (Fig. 10) are connected to the opposite sides of a d.p.d.t. switch with connecting wires of equal length. Before closing the circuit, introduce a large resistance into it by means of the rheostat r_1 for the purpose of protecting the milliammeter and the resistance box from too large a current. The key K is then closed, and the circuit completed

first through X and then through R. The resistance in R is now adjusted until the current in the circuit is the same regardless of whether the circuit includes X or R. Then the rheostats r_1 and r_2 are adjusted until the current in the circuit is near the upper limit tolerated by the highest resistance coil inserted into R (see Intr. 7) and the ammeter pointer is exactly on a division line. The larger ammeter deflection and the position of the pointer over a line enable the making of a more accurate adjustment of R for equality with X. The final comparison should be made by taking the observations for equality in quick succession. In case the resistance steps in R are too large for the obtaining of an exact balance, two ammeter readings are taken—one with R larger by the smallest possible amount than X, and one with R smaller. The value of R that would produce equality of deflection is then calculated by interpolation.

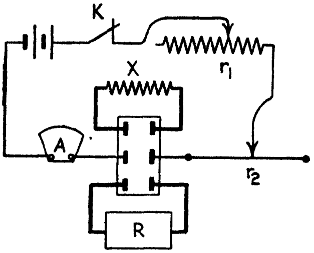


FIG. 10.

The measurement of resistance by this method, omitting details, consists of the substitution of an equal known resistance for the unknown. The unknown resistance

$$X = R \text{ ohms.} \tag{1}$$

To insure against errors due to poor contacts in the switch, the resistances X and R are interchanged, and their equality redetermined. The average of the two determinations gives the magnitude of X.

EXPERIMENT 5

POTENTIAL DIFFERENCE ALONG A CIRCUIT

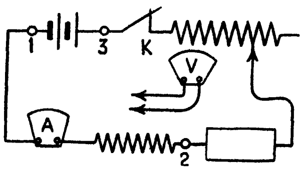


FIG. 11.

Apparatus.—A coil of known and a coil of unknown resistance (10 to 20 ohms each); galvanometer voltmeter (constructed in Exp. 2 and represented as a voltmeter in Fig. 11); milliammeter; rheostat.

Method and Manipulation.—After assuring oneself that the current will not exceed the allowable limit, connect the two

resistances, the milliammeter, and the rheostat in series with the battery, as shown in Fig. 11; then increase the current as far as the rheostat permits without crossing the allowable limit.

Measure the potential difference across the binding posts 1–2, 2–3, and 1–3. Also, after measuring the potential difference between 1–3, which is the total potential difference in the external part of the circuit, open the circuit at K, and observe how much the opening of the switch changes the voltmeter reading. The voltmeter leads should preferably be provided with plug-in prongs, and the binding posts with the proper receptacles for them. However, the leads can be held in hand while the contacts are being made with the binding posts.

Show from the observations (1) that in the external part of the circuit the sum of the two potential differences equals the total measured potential difference and (2) that the potential difference in the external circuit is practically equal to the e.m.f. of the battery.

Questions.

1. Why is the galvanometer voltmeter used in place of the commercial voltmeter?
2. Why is the total potential difference in the external circuit practically equal to the e.m.f. of the battery?
3. Why does the voltmeter measure the e.m.f. of the battery?

EXPERIMENT 6

LOW RESISTANCE. FALL-OF-POTENTIAL METHOD

Apparatus.—The apparatus of Exp. 5, except that, for greater convenience and accuracy, a d.p.d.t. switch is employed for quickly transferring the high-resistance voltmeter from one resistance to the other; the voltmeter is connected to measure the potential differences only across the two resistances which are to be compared, as shown in Fig. 12.

Method and Manipulation.—The connections are made so that the voltmeter deflects in the proper direction for both potential differences. This requires the leads to one of the resistances to be crossed, as shown in Fig. 12. The current strength is made as large as the standard resistance permits, if that is required to

obtain large deflections in the voltmeter. Then from Ohm's law the current in the main circuit

$$I = \frac{E_X}{X} = \frac{E_R}{R},$$

from which

$$X = \frac{E_X}{E_R} R \text{ ohms.} \tag{1}$$

This method compares the unknown resistance with the known.

To reduce errors:

1. The observations should be taken in quick succession to insure constancy of the current in the main circuit. This constancy is a condition required by the experiment.

2. The resistance of the voltmeter should be high enough so that it diverts only a negligible portion of the whole current. Any diversion of the current diminishes the potential difference across the resistance spanned by the voltmeter and therefore the voltmeter reading. If the two resistances X and R are nearly equal, both potential differences are diminished equally, so that no appreciable error is introduced in the determination of X even though the voltmeter may take an appreciable current. It is desirable therefore to make a preliminary determination of X and then to take the final observations with R nearly equal to X. This method is rarely used except for the measurement of very low resistances carrying large currents, where this source of error is negligible even with a commercial voltmeter.

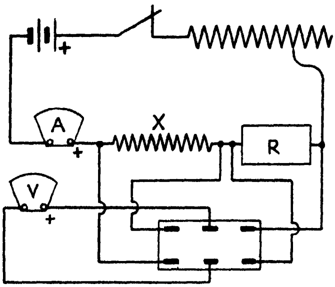


FIG. 12.

EXPERIMENT 7

RESISTANCE OF A CIRCUIT AND E.M.F. OF A BATTERY. OHM'S METHOD

Apparatus.—A battery of two cells in series with a milliammeter, a rheostat, and a resistance box.

Method and Manipulation.—After a protecting resistance is inserted into the rheostat, the circuit is closed and the rheostat is adjusted to give a nearly full-scale reading I_1 in the milliammeter. This is the circuit of which the resistance r is to be determined. Now insert into the resistance box such a resistance R which reduces the current to about one-half the magnitude of I_1. The resistance in the circuit then is $r + R$; and the current, I_2. From Ohm's law

$$I_1 = \frac{E}{r}, \qquad I_2 = \frac{E}{r + R},$$

whence

$$\frac{I_1}{I_2} = \frac{r + R}{r}.$$

Then, by subtracting one unit from each side of the equation and transposing, the resistance of the circuit is shown to be

$$r = \frac{I_2}{I_1 - I_2} R \text{ ohms}; \tag{1}$$

and the e.m.f. of the battery,

$$E = rI_1 \text{ volts}. \tag{2}$$

EXPERIMENT 8

HORIZONTAL INTENSITY OF THE EARTH'S MAGNETIC FIELD AND POLE STRENGTH OF A BAR MAGNET. METHOD OF GAUSS

This absolute method for measuring the horizontal intensity of the earth's magnetic field consists of two independent determinations. The magnitude of M/H is calculated from the deflection produced on a short magnetic needle by a bar magnet, and that of MH is determined from the time of vibration of this same bar magnet in the earth's magnetic field. Let $M/H = A$, and $MH = B$. Then

$$H = \sqrt{\frac{B}{A}}. \tag{1}$$

M = magnetic moment of the bar magnet,
H = the horizontal intensity of the earth's magnetic field.

1. Determination of MH

Apparatus.—The magnet to be used in the determination of M/H; an unspun-silk fiber attached to an aluminum saddle for suspending the magnet and hung in a box for excluding air currents; a circular scale; a watch.

Method.—The equation for the time of a double oscillation of a bar magnet suspended by a fiber of negligible torque is, when vibrating in a small arc,

$$t = 2\pi\sqrt{\frac{I_0}{MH}},$$

from which

$$MH = \frac{4\pi^2 I_0}{t^2} = B, \tag{2}$$

where I_0 is the moment of inertia of the magnet which can be calculated, t is the time of a double oscillation which must be determined experimentally.

Manipulation.—All magnetic materials on the table are moved as far as possible from the end of the table where H is to be determined. This requires the Edison battery to be placed on the floor and the milliammeter in the location where it is to be used with Exp. 9. The magnetic field in a modern laboratory is greatly modified in both intensity and direction by iron beams used in the construction of the building and by magnetic materials within the laboratory. The horizontal component H of this modified earth's field also differs from point to point within the room, and, therefore, a determination of its intensity must be made for a specific location and elevation.

The magnet is adjusted in the suspended saddle until it hangs in a horizontal position, and then the box is turned until one end of the magnet, when at rest, is directly above a line on the circular scale. The box cover is closed, and the magnet, after it is freed from pendular motion, is set into vibration by means of magnetic material through a total arc of not more than 30 deg.

The time of the double vibration may be determined as follows: One observer taps the table at the instant when the vibrating magnet is passing the center of its arc of vibration from left to right. This is repeated for about 300 seconds. The second

observer records the time of day for the passages to within 0.1 seconds. The difference, for example, between the first and the sixteenth observation gives the period for 15 double vibrations. The difference between the second and the seventeenth gives the same period. In this manner at least five independent values for the same number of vibrations is obtained for the purpose of reducing the experimental error.

Since the equation for MH contains the square of the time of vibration, the time must be determined with twice the percentage accuracy desired in the value of MH.

The magnet used is usually a cylinder, the moment of inertia of which, when rotating on an axis perpendicular to its center, can be calculated from $I_0 = M'\left(\frac{L_1^2}{12} + \frac{R^2}{4}\right)$, where L_1 is the length of the cylinder, R its radius, and M' its mass.

Since the expression for the time of vibration is correct for only very small angles, the comparatively large angles necessarily used require the making of a small correction to the observed time. When t is the observed time and θ the average complete arc of vibration, the corrected time

$$t_0 = t\left(1 - \tfrac{1}{4}\sin^2\frac{\theta}{4}\right).$$

When the average arc of vibration is 20 deg, this correction reduces the observed time of vibration by 0.19 per cent.

2. DETERMINATION OF M/H

Apparatus.—Simple pivot-needle magnetometer combined in one instrument with a tangent galvanometer. The magnetometer part of the instrument consists of a support with the pivot-needle compass, a horizontal linear scale, the bar magnet for which MH has been determined, a movable saddle for supporting the magnet, and an aligning device. The tangent galvanometer part, to be used in Exp. 9, consists of a coil of known number of turns and large diameter. The plane of the coil is in the vertical axis of the needle and perpendicular to the linear scale. The magnetometer and galvanometer are both required for measuring the strength of the electric current by the absolute method, and it is desirable

that both Exp. 8 and Exp. 9 be performed in the same laboratory period.

Method.—Figure 13 shows the bar magnet deflecting the magnetic needle. The distance between the magnet and the needle is relatively shortened, and the needle greatly enlarged. The distance between a pole of the magnet and the pivot of the

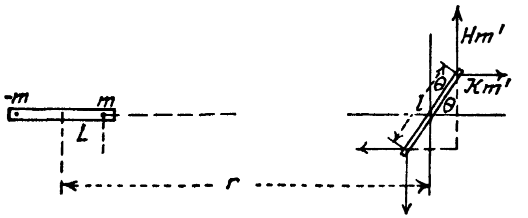


FIG. 13.

needle is $d = r \mp L$. The intensity of the field at the pivot of the needle, due to the bar magnet, is

$$\mathcal{H} = \frac{m}{(r-L)^2} - \frac{m}{(r+L)^2} = \frac{4rLm}{(r^2-L^2)^2} = \frac{2rM}{(r^2-L^2)^2}$$

Since the needle is short, the intensity of this field at the poles of the needle is practically that at the pivot. Its direction is also practically parallel to the line of the axis of the magnet. When the deflected needle is at rest, the torque due to the magnet acting on the needle in one direction is equal to that due to the horizontal component H of the earth's field acting on the needle in the opposite direction. Then the torque

$$L = \mathcal{H}m'l \cos\theta = \frac{2rM}{(r^2-L^2)^2}m'l\cos\theta = Hm'l\sin\theta,$$

from which

$$\frac{M}{H} = \frac{(r^2-L^2)^2}{2r}\tan\theta = A. \tag{3}$$

Manipulation.—The magnetometer is placed with its magnetic needle in the exact location for which MH was measured. The horizontal linear scale (Fig. 14) is adjusted until its center is directly under the pivot of the magnetic needle and the scale is perpendicular to the plane of the galvanometer coil. The

magnetometer then is turned on its vertical axis until the horizontal axis of the magnetic needle is in the plane of the galvanometer coil. The compass box, after being centered, is turned until the two null points on its circular scale are in line with the linear scale. The aluminum double pointer attached and at right angles

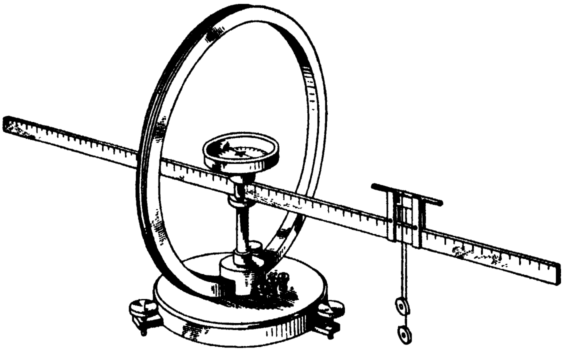


FIG. 14.

to the needle should then have its ends approximately over the null points. The movable saddle of proper height to raise the bar magnet to the level of the magnetic needle is placed on the scale and moved with the magnet into a position where the needle deflects about 20 deg. The aligning device is then hung over

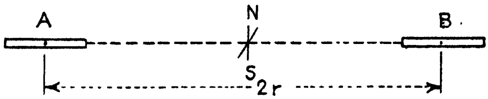


FIG. 15.

the center of the magnet, and the magnet is moved until the line of the aligning device shows the center of the magnet at *A* (Fig. 15) to be an exact whole number of centimeters from the pivot of the magnetic needle. The readings are then taken at both ends of the pointer, avoiding errors due to parallax (see Chap. VI, Intr. 5). If the instrument were perfect, one deflection and one measurement of the distance would suffice. But because the pivot of the needle may not be set exactly at the center of the circular scale, it is necessary to take the average of the two deflections as observed at the extremities of the aluminum pointer. Also, since the point poles may not be at the same distance from

the ends of the bar magnet, it is necessary to take deflections in the reverse direction after turning the bar magnet end to end. Also, because the linear scale may not have its center, from which the distance of the magnet from the needle is measured, placed exactly under the pivot of the needle, it is necessary to take a similar set of observation in position B (Fig. 15) at an equal distance on the other side of the compass needle.

It is not necessary to record the null points, because deflections are taken on both sides of each null point, and the sum of the two observed deflections is twice the average deflection regardless of the exact position of the null reading.

Because of friction at the pivot, the needle may not come exactly to its true position of rest. A light tapping on the compass box brings the needle closer to its correct position.

The readings of the deflected needle may be recorded as follows:

READINGS

Position of magnet	East pointer	West pointer
West { original	23.1° ± 1	24.7° ± 1
West { reversed	22.0	20.9
Double deflection	45.1 ± 1	45.6 ± 1
East { original	23.2°	25.1°
East { reversed	21.5	20.0
Double deflection	44.7	45.1

The estimated error of 0.1° in a single reading appears in the double deflection as it would appear if a single deflection were taken; likewise, in the average of the four double deflections, because the differences in their magnitudes are due to other causes than chance errors. The foregoing illustrative observations then make

$$2\theta = 45.1° \pm 1, \qquad \text{and} \qquad \theta = 22.55° \pm 5.$$

The distance L [Eq. (3)] is obtained by placing a small short-needle compass beside a centimeter division of a wooden scale. The scale is turned until the centimeter division line aligns with

the needle of the compass. The magnet is then moved on top of the scale and close to the compass until the needle again aligns with the centimeter division. The distance of the point pole from the end of the magnet then is the distance that the end extends beyond the centimeter division. Since L^2 is only a small fraction of r^2 and is to be subtracted from it, the magnitude of L, which is one-half the distance between the point poles, is determined with sufficient accuracy by this method.

Calculations.—1. The magnitude of H is determined by substituting the values of A and B in Eq. (1), where

$$H = \sqrt{\frac{B}{A}} \text{ oersteds.} \tag{4}$$

2. The magnitude of the pole strength of the bar magnet is determined from Eqs. (2), (3) and from the magnetic moment $M = 2Lm$. Since

$$\frac{M}{H} = A \qquad \text{and} \qquad MH = B, \qquad M = \sqrt{AB}.$$

Then the pole strength

$$m = \frac{M}{2L} = \frac{\sqrt{AB}}{2L} \text{ units.} \tag{5}$$

EXPERIMENT 9

ABSOLUTE MEASUREMENT OF ELECTRIC CURRENT. TANGENT-GALVANOMETER METHOD

Apparatus.—The tangent galvanometer adjusted in Exp. 8, where H was determined for use with this absolute method of current measurement; commutator; nonmagnetic rheostat; battery; milliammeter; switch. The milliammeter is included only for the purpose of checking the determined magnitude of the current.

Method.—In the undeflected position of the compass needle in an adjusted tangent galvanometer, the axis of the needle is in the plane of the coil. Then the horizontal component of the earth's magnetic field exerts forces on the point poles urging them in the north and south direction, while the field due to the current

in the coil exerts forces urging the poles at right angles to the plane of the coil. In this position the forces due to H exert no torque, and those due to the current exert their maximum torque. The coil then turns until the two torques are equal, as represented in Fig. 16. Here the length of the needle is greatly enlarged compared to the represented diameter of the coil. The two equal torques are

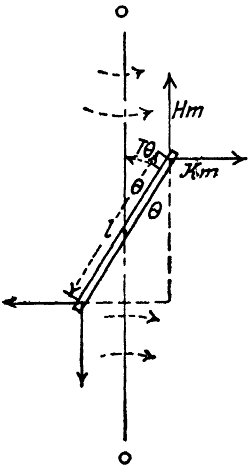


FIG. 16.

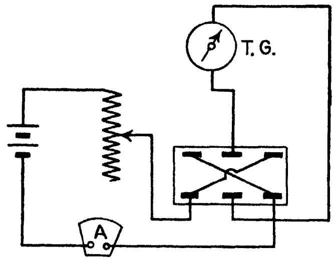


FIG. 17.

$$L = \mathcal{H}m \cdot l \cos \theta = Hm \cdot l \sin \theta.$$

But

$$\mathcal{H} = \frac{lI'}{r^2} = \frac{2\pi r N I'}{r^2} = \frac{\pi N I}{5r} \text{ oersteds.}$$

Then

$$\frac{\pi N I}{5r} \cos \theta = H \sin \theta,$$

and

$$I = \frac{5Hr}{\pi N} \tan \theta \text{ amp.} \tag{1}$$

Manipulation.—The instruments are connected as shown in Fig. 17. The battery is placed on the floor, and the rheostat and milliammeter on the distant end of the table. The wires leading to the galvanometer should be twisted together to limit their magnetic effect on the galvanometer. The current is adjusted to give a reasonably large deflection in both the instruments. The strength of the current is calculated, using Eq. (1), from the average of three double deflections. The calculated magnitude

should equal that given by the milliammeter to within the limits of experimental error.

Question.—What is meant by an absolute method of measurement?

EXPERIMENT 10

I. ABSOLUTE MEASUREMENT OF POTENTIAL DIFFERENCE AND PROOF OF OHM'S LAW. CALORIMETER METHOD
II. ABSOLUTE MEASUREMENT OF RESISTANCE

Apparatus.—Electric calorimeter whose coil has a resistance of about 1 ohm; ammeter with 5-amp range; battery of six Edison cells; two thermometers, one of which preferably should be a calibrated thermocouple, but in either case the instrument to be immersed must give temperatures to within 0.01°C.; rheostat; slide-wire resistance; switch; watch; voltmeter, included only for comparing with its reading the determined magnitude of the potential difference.

An electric calorimeter differs from an ordinary calorimeter in having a resistance coil immersed in the liquid. It is used for measuring the heating effect of a current and therefore the electric energy supplied to and expended in the coil.

Method.—The potential difference between two points is measured by the energy expended in moving a unit quantity of electric charge from one point to the other. All this energy is transformed into heat within the conductor. The amount of this heat H, in calories, can be measured by means of the calorimeter; whence, from the definition of potential difference,

$$w_J = QE \text{ joules,}$$

from which

$$E = \frac{w_J}{Q} = \frac{J_J H}{It} = \frac{J_J(MS + M'S' + K)(T' - T)}{It} \text{ volts} \quad (1)$$

where J = the energy in joules required to heat 1-g of water through 1°C. and MS, $M'S'$, K = the water equivalents of the water, the calorimeter, and the thermometer.

The specific heat S' of brass is 0.095, and the water equivalent of the mercury thermometer is approximately 1.0.

The specific heat of water varies with the temperature; therefore the amount of energy required to raise 1-g of water 1°C. varies:

$$
\begin{aligned}
\text{At } 15°\text{C.}, &\quad J_J = 4.189 \text{ joules},\\
20°\text{C.}, &\quad J_J = 4.179 \text{ joules},\\
25°\text{C.}, &\quad J_J = 4.173 \text{ joules},\\
30°\text{C.}, &\quad J_J = 4.171 \text{ joules}.
\end{aligned}
$$

Manipulation.—1. The total mass M' of the water-containing beaker together with the heating coil and its supports and the metal part of the stirrer should be furnished with the calorimeter. Weigh the calorimeter to within 0.1-g when it is empty and again after it is filled with water to within 2 cm from the top and after the temperature of the water has been adjusted to about 15°C

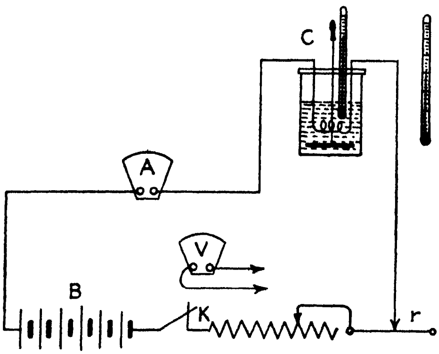


FIG. 18.

below that of the room. The difference in weight determines the mass of the water. For an ordinary laboratory determination, tap water may be used; but the water-containing beaker must be well insulated.

2. Connect the calorimeter into an electric circuit, as shown in Fig. 18, *taking care not to energize the heating coil without the coil being immersed in the water.* Adjust the current to approximately 5 amp, and then immediately open the circuit unless it is necessary to heat the water to the proper starting temperature. In the larger calorimeter, holding about 250-g of water, the starting temperature should be about 5°C. below that of the room; while with the smaller calorimeter, holding about 75-g, this temperature difference should be at least 7°C. Adjust the position of the immersed thermometer or thermocouple so that its bulb or junction is completely covered with water and does not touch the heating coil or interfere with the stirrer. After gently stirring

the water for several minutes and observing that the thermometer has acquired the temperature of the water, all is ready for the experiment to begin.

3. One observer operates the stirrer and reads the initial and final temperatures of the water to within 0.01°C. The other closes and opens the circuit and records the time of day to within 0.1 seconds for each of the two events, records the room temperature and tells the first observer at what temperature he should signal for the circuit to be opened, records the ammeter reading, and keeps the current constant by adjusting the slide-wire resistance r.

The circuit is opened when the temperature of the water is almost as far above that of the room as the initial temperature was below. The final temperature reading, however, is not taken until it reaches its maximum. If the foregoing directions are followed, no correction is required for a gain or loss of heat by radiation and conduction.

After the final reading has been taken, the circuit is again closed, and, after adjusting the current to its average value in the experiment, the potential difference across the calorimeter is measured with a voltmeter. This reading is taken only for the purpose of comparing it with that determined by the experiment. This potential difference across the binding posts is practically that across the immersed coil.

The experiment is then repeated using a current of 3.5 in place of 5 amp. On account of the slower rate of heating, it is desirable not to have the initial temperature more than 3 or 4°C. below that of the room.

Calculations and Proof of Ohm's Law.—1. The magnitude of the potential difference is calculated for each of the two cases from the observations by substituting in Eq. (1). Compare each determined potential difference with that measured directly.

2. The determined potential differences are plotted against the currents. If a straight line drawn from the origin passes through both the determined points,

$$\frac{E}{I} = \text{constant.}$$

This is Ohm's law, the constant being the resistance of the heating coil.

3. The data of the experiment may be used to determine R directly in terms of the energy expended and the current.

$$w_J = QE = RI^2t = J_JH \text{ joules},$$

from which

$$R = \frac{J_JH}{I^2t}.$$

This expression is that for E divided by I [see Eq. (1)]. Calculate R from each of the two determined values of E, and compare the average value with that measured by comparison methods.

NOTE.—The student should appreciate the fact that he has measured the potential difference in terms of the work in joules required to transfer 1 coulomb of electric charge through the conductor and, therefore, that he has measured the potential difference in terms of those quantities in which it is primarily defined. He should know also that such a method is called an *absolute method* of measurement.

EXPERIMENT 11

THE RESISTANCE OF A UNIFORM WIRE IS PROPORTIONAL TO ITS LENGTH

Apparatus.—A portable galvanometer with a high resistance R in series; Wheatstone-bridge wire; rheostat; milliammeter; etc.

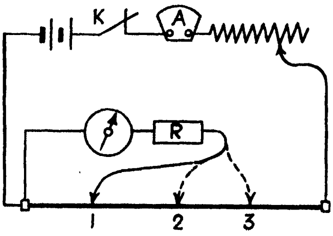


FIG. 19.

Method and Manipulation.—A resistance of about 2000 ohms is placed in series with the portable galvanometer to make of it a high-resistance galvanometer or a voltmeter. This high-resistance galvanometer is connected, as shown in Fig. 19, to span different lengths of the Wheatstone-bridge wire which is in series with a battery and a rheostat. A current of 50 ma in the battery circuit usually is sufficient, but a larger current may be used if required and the carrying capacity of the wire permits.

The high-resistance galvanometer need not be standardized to read volts. It is necessary only to know that the galvanometer deflections are proportional to the current—near enough for this

experiment—and that therefore they are proportional to the potential differences across the sections of the wire spanned by the galvanometer. Expressed mathematically,

$$\frac{d_1}{d_2} \cong \frac{i_1}{i_2} = \frac{e_1}{e_2}. \tag{1}$$

The galvanometer deflections are taken in quick succession from spanned lengths of 30, 60, and 90 cm. These deflections are plotted against the lengths. If they align with the origin of the plot, it is shown that the deflections vary as the lengths of wire spanned, *i.e.*,

$$\frac{d_1}{d_2} = \frac{l_1}{l_2}. \tag{2}$$

It follows from Eqs. (1), (2) that

$$\frac{e_1}{e_2} = \frac{l_1}{l_2}. \tag{3}$$

But since the current I in all the sections spanned is the same current, it follows from Ohm's law $e = rI$ that $e \propto r$, which, otherwise expressed, is

$$\frac{e_1}{e_2} = \frac{r_1}{r_2}. \tag{4}$$

Then from Eqs. (3), (4)

$$\frac{r_1}{r_2} = \frac{l_1}{l_2}, \qquad \text{or} \qquad r \propto l. \tag{5}$$

Note.—The assumed conditions require that the current in the Wheatstone-bridge wire remains constant during the whole experiment. Why is a high-resistance galvanometer used in place of a commercial voltmeter?

EXPERIMENT 12

RESISTANCE. WHEATSTONE WIRE-BRIDGE METHOD

Apparatus.—A Wheatstone wire bridge and accessories; the resistance to be measured.

Method.—The connections are made so that the resistance X to be measured and the known resistance R form two arms of the

Wheatstone bridge, while the uniform wire L_XL_R forms the other two arms, as shown in Fig. 20. The resistance of the connecting bars must be negligible compared with the resistances X and R. The slide S is moved along the wire until the potential at S is that at P. No current then energizes the galvanometer, and the bridge is said to be *balanced*. Then the potential difference

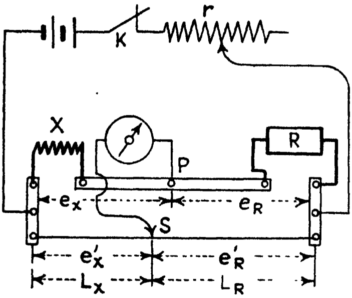


FIG. 20.

$$e_X = e'_X, \qquad \text{and} \qquad e_R = e'_R.$$

From which

$$\frac{e_X}{e_R} = \frac{e'_X}{e'_R}. \tag{1}$$

But since the same current energizes both X and R, it follows from Ohm's law that

$$\frac{X}{R} = \frac{e_X}{e_R}. \tag{2}$$

Similarly,

$$\frac{L_X}{L_R} = \frac{e'_X}{e'_R}. \tag{3}$$

Then from Eqs. (1), (2), (3)

$$\frac{X}{R} = \frac{L_X}{L_R},$$

and

$$X = \frac{L_X}{L_R} R \text{ ohms.} \tag{4}$$

Manipulation.—The current, at first, is reduced to the minimum by means of the high-resistance rheostat, as a protection to the galvanometer during the period of search for an approximate value of the trial balance. To obtain this trial balance, some reasonable resistance is placed in R, and the trial-balance point is found, from which an approximate value of the resistance X is calculated by Eq. (4). For the final balance, the resistance in R is made nearly equal to X, in order that the balance point may be near the center of the bridge wire. To increase sensitivity, the resistance in the rheostat r is diminished until moving the contact

point S 1 mm on the bridge wire produces a change of at least 1 mm in the galvanometer deflection. The bridge is then sensitive enough to make the balance as accurately as it is possible to make it with the apparatus.

Since the wire may not be uniform, and on account of possible errors due to poor contacts, X and R are interchanged, and the value of the resistance X redetermined. The average of the two values is taken as the magnitude of the resistance X.

NOTES.—Read Intr. 7 for instructions on the use of the resistance box. If a thermoelectric current deflects the galvanometer, the null reading is found with the switch K open and the contact at S closed, and, in case self-inductance is sufficiently large to produce disturbing throws, the switch K should be closed before contact is made at S.

EXPERIMENT 13

RESISTANCE. WHEATSTONE BOX-BRIDGE METHOD

The Wheatstone box bridge is a resistance box with keys and binding posts so placed as to make a convenient arrangement for use as a Wheatstone bridge. A diagram of one form of the bridge

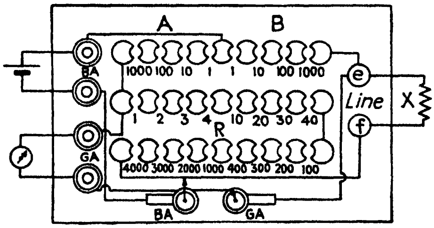


FIG. 21.

is shown in Fig. 21. The two sets of coils A and B take the place of the wire in the bridge of Exp. 12 and are called the *ratio arms,* or *bridge arms,* while the coils of the set R are usually designated as the rheostat, or *rheostat arm.* The resistance X to be measured is connected to the binding posts e and f, and a balance of the bridge is obtained as in Exp. 12. However, after the proper ratio of A to B to be used has been determined, it is the resistance in R that is varied in place of the arm ratio.

At balance

$$\frac{R}{X} = \frac{A}{B},$$

whence

$$X = \frac{B}{A}R \text{ ohms.} \tag{1}$$

If the value of the resistance to be measured is known approximately, the ratio of the resistances in the ratio arms is made such, whenever possible, as to require at least 1000 ohms in the rheostat arm to produce a balance of the bridge. Otherwise, a preliminary measurement of X is made by means of any reasonable ratio. This approximate value of X then enables the proper ratio, required to make $R > 1000$, to be calculated. Since one ohm is the smallest resistance in the rheostat arm, the adjustment of the bridge can be made only to within that limit. However, if the resistance in the arm is at least 1000 ohms, the adjustment is made to within 0.1 per cent. This accuracy is sufficient for most measurements, and the coils of most bridges and resistance boxes are adjusted only to within that limit. Better bridges and boxes have their resistances adjusted to 0.01 per cent (see also Intr. 7).

It is preferable to use only one coil in each ratio arm and to avoid as far as possible the use of the 1-ohm ratio coils.

The galvanometer circuit should be closed first to test for thermoelectric current in the circuit. If any deflection of the galvanometer is observed, the bridge must be balanced to this deflected position and not to the open-circuit null point.

If the circuit contains self-inductance, first close the battery circuit and then the galvanometer circuit. This is the usual procedure after the test for thermoelectric current has been made.

The circuit should be closed for a period no longer than is necessary, to avoid heating the coils and thereby changing their resistance. The current-carrying capacity of the coils in an ordinary resistance box with wooden spools is about 0.25 watts. When necessary, 0.5 watts may be used for an instant only.

When measuring resistances of less than 7 ohms, it can be shown from the foregoing that the carrying capacity of the 1-ohm arm coil is exceeded when a 2-volt battery is used with the bridge. An extra resistance is then introduced in series with the battery unless the manufacturer has made the carrying capacity of that coil larger than that of the others.

In the measurement of small resistances, the resistance of the connecting wires may not be negligible. This resistance then must be determined separately.

Make at least the first of the following measurements:

1. Resistance of two coils of wire separately, in series, and in parallel.
2. The resistance per foot of a copper wire of known gauge, at 20°C. Calculate from the result the specific resistance of copper.
3. The resistance of a high-resistance coil at 20°C.
4. The resistance of a carbon megohm.
5. The resistance of the human body with dry and then with moist contacts. For moist contact insert each thumb into a tube containing salt water.
6. A resistance containing a small e.m.f.
7. A resistance with self-inductance.

EXPERIMENT 14

ELECTROMOTIVE FORCE OF INDIVIDUAL CELLS AND OF THE CELLS IN SERIES

Apparatus.—Two storage cells; a voltmeter.

Method and Manipulation.—Determine the e.m.f. of each of the two cells and then of the two in series, by attaching the voltmeter directly to the cells. Show that $E_{1+2} = E_1 + E_2$.

The voltmeter measures only the potential difference across its binding posts or between the binding posts to which it is connected by leads of negligible resistance. The RI drop within the battery is not measured, and therefore the readings given by the voltmeter are too small by that amount. If the resistance of the voltmeter is large, this error is negligibly small unless the resistance of the cell is abnormally large. The resistance of a storage cell usually is less than 0.01 ohm.

Note.—The e.m.f. of the cells in parallel should not be measured in the laboratory, partly because of the danger of short-circuiting the cells, and because, if the e.m.fs. are not exactly alike, one cell charges the other through a negligible resistance.

EXPERIMENT 15

ELECTROMOTIVE FORCE OF A VOLTAIC CELL. POTENTIOMETER METHOD

Apparatus.—The uniform high-resistance wire and the sliding contact of a Wheatstone wire bridge; two storage cells for energiz-

ing the main circuit; a rheostat; portable galvanometer; a high resistance HR preferably in the form of a dial resistance box; the two cells whose e.m.fs. are to be compared.

Method.—The current from the main battery B produces a potential gradient along the uniform wire from Y to Z (Fig. 22). Since the potential at Y is higher than that at C, currents flow from Y to C in both branches—in one through the wire, and in the other through the galvanometer. If there is a voltaic cell

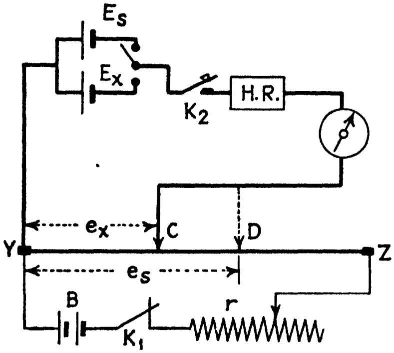


FIG. 22.

with an *opposing* e.m.f. in the galvanometer branch, the current through the galvanometer will be zero when the potential difference e_x between Y and C on the wire is equal to the opposing e.m.f. E_x of the cell. This condition is attained by moving the contact slide C. The length of the wire l_x between Y and C is recorded. The cell E_x is then replaced by E_s, whose e.m.f. is known. The balance is again obtained, and the spanned length l_s of the wire is recorded. The potential difference e_s between Y and D then is equal to the known e.m.f. E_s of the cell.

Then

$$\frac{E_x}{E_s} = \frac{e_x}{e_s} = \frac{l_x}{l_s},$$

from which

$$E_x = \frac{l_x}{l_s}E_s.$$

E_s may be a storage cell whose e.m.f. has been measured with a voltmeter, or it may be a standard cell whose e.m.f. is known,

Manipulation.—The high resistance HR is introduced into the resistance box to diminish the potential sensitivity of the galvanometer; but this may finally be entirely removed if the attaining of a desired sensitivity requires it. The open-contact key K_2 should be omitted if the contact slide on the bridge wire normally keeps the galvanometer circuit open. The resistance in the rheostat r should be adjusted until the longer of the lengths l_x and l_s includes nearly the whole length of the wire. The sensitiveness of the galvanometer at final balance should be high enough to detect a displacement of 0.1 scale division on the wire.

Questions.

1. The e.m.f. of the battery B must necessarily be greater than that of either of the cells whose e.m.f. is being balanced. Why?
2. How does a high resistance in the galvanometer branch affect the measurement?
3. In what respect is this method an improvement over a direct measurement with a voltmeter?
4. Can a potential difference between any two points in a circuit be measured by this method?

EXPERIMENT 16

ADJUSTMENT OF THE MOVING-COIL GALVANOMETER

The moving-coil galvanometer is described in Intr. 1. The upper suspension of such an instrument is fragile and easily broken even by jarring. The galvanometer, therefore, should not be moved without the coil first being lowered for support on the cylinder C (Fig. 23). The form of the galvanometer here described is that now in common use for ordinary laboratory and industrial purposes.

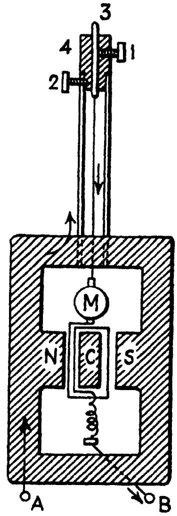


Fig. 23.

1. Adjustment of Coil and Leveling of Galvanometer.—*a.* Remove the reading telescope and scale with the supporting rod and then the front cover of the galvanometer.

b. Remove or attach the damping rectangle, depending on the purpose for which the galvanometer is to be used. Avoid touching the coil with the bare hands as far as is possible, as touching may contaminate it with magnetic dust particles.

c. Make the preliminary leveling of the instrument.

d. Holding the rod 3 (Fig. 23) by hand, loosen screw 1; and, while watching the coil, raise the rod carefully until the coil hangs freely and clears the cylinder equally on top and bottom. Then lightly tighten screw 1 to hold rod 3, remembering that the delicate screw threads are easily stripped and that dropping the rod breaks the suspension.

e. Level the galvanometer until the coil hangs symmetrically with respect to the cylinder and the axis of the cylinder coincides with that of the coil. The plane of the coil then is nearly parallel to the radial magnetic lines of force in all deflected positions, and, therefore, the deflections on a circular scale are proportional to the current.

f. Replace the front cover.

g. Being certain that screw 1 is holding rod 3, loosen screw 2, and then turn lug 4 until the plane of the coil is parallel to that of the face of the instrument. If the coil oscillates on its vertical axis, short-circuit the coil through the binding posts *AB*. This adjustment is aided by noting whether, when the image of the observing eye is seen in the mirror, the line between the eye and the mirror *M* is perpendicular to the face of the galvanometer.

2. Adjustment of the Reading Telescope and Scale.—The telescope and scale are usually clamped to a rod (Fig. 24) which has provision for an adjustable attachment to the galvanometer. The reading telescope consists of three tubes. The first tube contains a lens, called the *objective*, which collects the light and forms a real image of the observed object at a point within the second tube. The second or *focusing tube* contains a crosshair which serves as the index line. In adjustment, this crosshair is in the plane of the real image of the observed object. The third tube, called the *eyepiece*, contains a lens which serves as a simple microscope for magnifying the real image and the crosshair.

a. Raise or lower the telescope and scale by means of the leveling screw *S* at the galvanometer end of the rod into a position where a line perpendicular to the mirror passes through a point halfway between the axis of the telescope and the reading scale. This adjustment often is most readily made by holding the eye on the level of the telescope and raising or lowering the supporting rod until the image of the scale is seen in the mirror.

b. Adjust the eyepiece until the crosshair is seen most distinctly.

c. Adjust the focusing tube until the image of the scale is seen most distinctly. The crosshair then should be in the plane of this image and free from parallax (see Intr. 5). In order to obtain this freedom from parallax and at the same time see the scale image most distinctly, it may be necessary to correct an error, which is often made, in focusing the eyepiece on the crosshair. Also, it is necessary for the protection of the eye against strain, if many readings are to be taken, to have these superposed images exactly in the plane of most distinct vision. This position can be

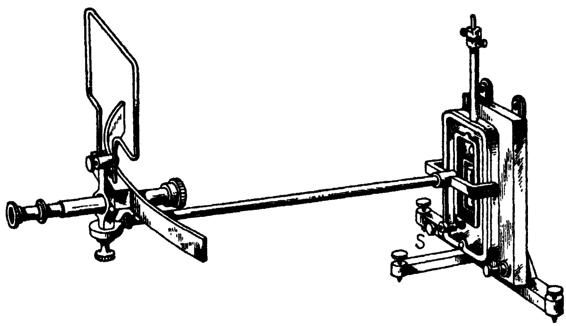


Fig. 24.

determined from the two positions of the focusing tube where the image of the scale begins to blur.

d. Observations with a reading telescope should always be taken with both eyes open.

e. The reading scale should be well illuminated; and, if a window is located directly across the room, a screen must be hung above the scale to prevent the diffused light from falling on the mirror and thereby, on reflection, obscuring the scale image.

f. If it is difficult to find the image of the scale:

(1) Point telescope at mirror.

(2) Check adjustment *a* for proper elevation of the telescope support.

(3) Adjust focusing tube until a luminosity appears. Center this luminosity, and adjust focus until the blur becomes a clear field. This field should contain a clear image of the scale.

(4) If the image of the scale is too high or too low in the luminous field, lower or raise the telescope by means of the supporting rod.

(5) To adjust the scale to zero reading, move it until the zero mark is directly above the axis of the telescope. Then turn the

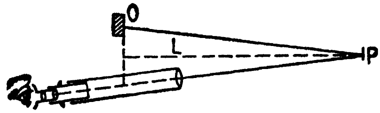


FIG. 25.

coil by means of lug 4 (Fig. 23), care being taken not to loosen the wrong screw, which would drop the suspension support and break the suspension. The final small adjustment is made by moving the scale sidewise. It must be remembered, however, that such an adjustment destroys the exact current proportionality of the deflections, because the scale, which should be perfectly circular, must, for such proportionality, also have its center of curvature in the axis of the mirror. For ordinary use, however, such small imperfections are tolerated.

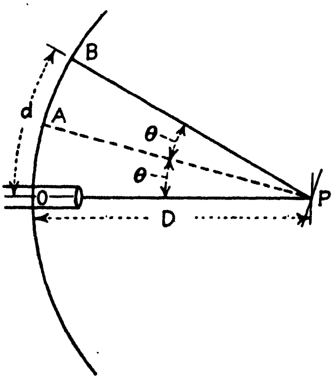


FIG. 26.

g. It is more convenient, though not capable of so high a degree of accuracy, to employ another kind of optical system than that just described. A tube holding a straight-filament lamp replaces the telescope. Either the image of the filament or the shadow of a wire through which the light from the lamp passes is focused on the scale after reflection from the galvanometer mirror. If the scale is translucent, readings may be taken from either side.

3. Measurement of Angles of Deflection.—When the null point of the scale appears on the crosshair of the telescope, a ray of light coming from the null point 0 of the scale (Fig. 25) is reflected from the mirror at the point *P*. The incident and reflected rays are in the same vertical plane perpendicular to the mirror. When the mirror is turned through the angle θ, this

vertical plane turns from PO (Fig. 26) to PA, so that a ray of light coming from B in a vertical plane BP is reflected in the vertical plane PO, making the angle $BPO = 2\theta$.

If d is the observed deflection OB, and D the perpendicular distance between the plane of the mirror and the plane of the scale, then

$$2\theta = \frac{d}{D} \text{ radians,}$$

from which on a circular scale the angle of deflection

$$\theta = \frac{d}{2D} \text{ radians.}$$

The distance D is also that from the axis of the coil (and fiber) to a point midway between the axis of the telescope and the center of the scale. This distance is measured accurately enough for ordinary purposes by means of a meter stick, the measurement being made from the center of the suspension tube to the plane of the scale.

4. Damping the Coil.—The galvanometer coil may be wound on an aluminum rectangle which damps the vibrations of the coil by generating an electric current within the rectangle at the expense of the energy of vibration of the coil. The coil, too, may be wound on a spindle which gives shape to the coil and, when removed, leaves the coil without a frame. Such a coil is not damped on open circuit, but on closed circuit the coil generates an e.m.f. which causes a degree of damping depending on the resistance in the circuit. The coil may be overdamped, critically damped, or underdamped by the introduction of the appropriate resistance into the circuit or by proper shunting. Such a coil, too, as in the instrument described, may be damped by attaching a copper rectangle to it. Coils of very high resistance may be underdamped even on closed circuit. Such coils may be brought to null reading without crossing it after a deflection by an appropriate motion of a magnet within an open coil which is included in the circuit or is attached temporarily as a shunt by means of a key.

5. Zero Shift.—Deflections and throws are always taken from left to right and the coil is not allowed to cross the null point to the left by more than a few millimeters; because if the coil is

not accurately centered in a radial magnetic field, a reversal of deflection causes the null-point reading to change on account of a directional change in the residual magnetism of the magnetic impurities within or on the coil.

NOTE.—This experiment is intended to acquaint the student with the sensitive moving-coil galvanometer and its adjustment. It is not necessary, at this time, for him to carry through all the operations outlined. However, he should adjust and properly focus the reading telescope and perform some qualitative experiments, such as *L* and *N* given in the list of qualitative experiments.

The galvanometer should be left in adjustment for future use, but the focusing of the scale may have to be repeated, because some students require other than normal optical accommodation.

EXPERIMENT 17

POWER EXPENDED IN A DIRECT-CURRENT CIRCUIT

Apparatus.—Four storage cells; ammeter; voltmeter; wattmeter (see Chap. VI, Intr. 4); rheostat.

Method and Manipulation.—Connect the apparatus as shown in Fig. 27, with the ammeter and the field coil of the wattmeter in a series circuit with the resistance *r*. The binding posts *AB*, between which the power consumed in the resistance *r* is to be measured, should preferably be provided with openings for plug-in prongs.

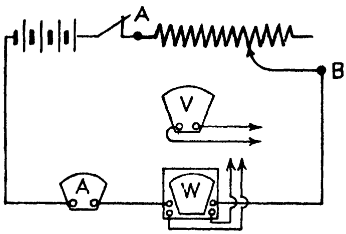


FIG. 27.

Adjust the rheostat to give a current of about 1.5 amp. Then measure the potential difference between the binding posts *AB* with the voltmeter and the power consumed in the part of the circuit between *A* and *B* by spanning these points with the potential leads of the wattmeter. Since

$$P = EI \text{ watts,}$$

the product of *E* and *I* in a direct-current circuit should equal the power measured directly with the wattmeter.

Special care must be taken not to exceed the rated current-carrying capacity of the field coil of the wattmeter.

EXPERIMENT 18

ALTERNATING-CURRENT SERIES CIRCUIT

Apparatus.—A series circuit MN (Fig. 28) consists of three sections R_0, L_0, C_0. The section R_0 should be practically non-reactive, so that its apparent resistance R_1 may have the same or nearly the same magnitude as the ohmic resistance r. The section L_0 is a choke coil and, therefore, has both self-inductance and ohmic resistance. The apparent resistance R_2 is larger than the ohmic resistance because the power expended in overcoming the hysterisis of the iron core and in the production of eddy currents is treated as though it were expended in an extra series resistance. The section C_0 contains a paper condenser and practically no ohmic resistance. The apparent resistance R_3 is due to treating the energy lost through the action of the absorbed charge and the leakage through the condenser and over its surface as though the energy were expended in a series resistance. The ammeter A measures the current in all the sections of the series circuit. The field coil of the wattmeter W is permanently in circuit; and its potential leads and those of the voltmeter end in plug-in prongs for quickly spanning any desired section of the circuit. In the case of the wattmeter, the prongs must be transposed if the deflection of the wattmeter is in the wrong direction. The adjusting rheostat S, the fuse F, and the switch K are also in series but outside the experimental series circuit. The energy is supplied by inserting a plug at H into an alternating-current socket supplying a 60-cycle e.m.f. of 115 volts.

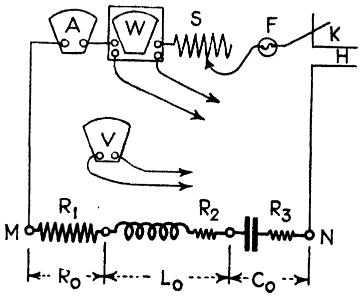


FIG. 28.

The convenient magnitudes of r, L, and C in the three sections R_0, L_0, C_0 are 140 ohms, 0.35 henrys, and 9 microfarads, respectively; and the ranges of the three alternating-current instruments, 1 amp., 150 volts, and 75 watts, respectively.

Manipulation and Observations.—With the potential leads of the wattmeter and voltmeter both disconnected, as shown in

Fig. 28, adjust the current in the circuit to 0.500 ± 1 amp by means of the rheostat S. This current magnitude is maintained throughout the whole experiment. If the current varies owing to fluctuation of line voltage, adjust S until the magnitude to which the current tends to return after erratic fluctuations has this value. Each voltmeter and wattmeter reading is then taken as quickly as possible and is recorded only if, after disconnecting the instrument, the ammeter in the main circuit shows that no change in the e.m.f. impressed on the circuit has taken place during the reading interval.

The ammeter reading changes, also, because of the inductive influence and the current shunted through the voltmeter and the potential coil of the wattmeter when these span any section of the circuit. These changes should be recorded for reference but not used, because of the difficulty in making properly the corrections for them. The current is not adjusted to its original value because such an adjustment would not correct the disturbing influence. The magnitude of the decrease or increase in the current does not indicate accurately the magnitude of the error introduced, and, when no correction is made for this source of error, the determined quantities do not check to within the usual tolerated experimental error.

1. Record the zero readings of all three reading instruments. They should all read zero.

2. Adjust current to 0.500 ± 1 amp unless the instructor directs the use of another magnitude.

3. Take the potential difference across each of the three sections and across all three together as described in the foregoing paragraphs. Repeat three times, and use the average value in the tabulation.

4. Take similar sets of power readings with the wattmeter.

Calculations.—Calculate and tabulate the quantities listed below for each of the three sections and for the three sections in series.

1. Apparent Resistance.—From the equation $P = RI^2$

$$R = \frac{P}{I^2} \text{ ohms,} \tag{1}$$

where R is the apparent resistance which normally is larger than

the ohmic resistance r, as already explained, except in branches in which energy is dissipated only in the normal heating of the conductor.

Tabulate the ohmic as well as the apparent resistances.

2. Impedance.—From the equation

$$I = \frac{E}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{E}{Z}$$

the impedance

$$Z = \frac{E}{I} \text{ ohms,} \tag{2}$$

where E and I are known. Determine the magnitude of Z for each of the four cases. In the nonreactive section R_0, the impedance should equal the apparent resistance.

3. Reactance.—The reactance X for each of the four cases is determined from the expression for impedance, where $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + X^2} = \frac{E}{I}$, from which the reactance

$$X = \sqrt{\left(\frac{E}{I}\right)^2 - R^2} \text{ ohms.} \tag{3}$$

4. Self-inductance.—In the inductive section, where the capacitance is negligible, the inductive reactance $X_L = \omega L$, from which

$$L = \frac{X_L}{\omega} \text{ henrys.} \tag{4}$$

5. Capacitance.—In the capacitive section, where the self-inductance is negligible, the capacitive reactance $X_c = 1/\omega C$, from which

$$C = \frac{1}{\omega X_c} \text{ farads, and } C_{mf} = \frac{10^6}{\omega X_c} \text{ microfarads.} \tag{5}$$

6. Power Factor.—From the equation for the power expended $P = EI \cos \theta$, the power factor

$$\cos \theta = \frac{P}{EI}. \tag{6}$$

7. Phase Difference.—The phase difference in degrees is determined in each case from the known magnitude of the power factor $\cos \theta$.

8. In-phase Component.—The in-phase component of the impressed e.m.f., in each case, is seen from the vector diagrams (Fig. 29) to be

$$E \cos \theta = RI. \tag{7}$$

Its magnitude may also be calculated from the power equation from which

$$E \cos \theta = \frac{P}{I}. \tag{8}$$

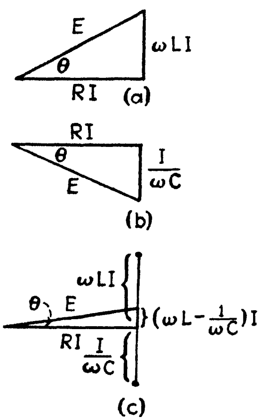


FIG. 29.

9. Out-of-phase Component.—The out-of-phase component of the impressed e.m.f. is the component 90 deg out of phase with the current and in the vector diagrams (Fig. 29) is seen to have the magnitude

$$E \sin \theta = \omega LI, \text{ or } \frac{I}{\omega C}, \text{ or } \left(L\omega - \frac{1}{\omega C}\right)I, \tag{9a}$$

depending on the nature of the reactance in the circuit.

The magnitude of $E \sin \theta$ may also be calculated from the known values of E and $E \cos \theta$. The vector diagram shows that

$$E \sin \theta = \sqrt{E^2 - (E \cos \theta)^2}. \tag{9b}$$

10. Table of Results.—Tabulate the observed and the nine calculated quantities for the four cases in the order described. However, include also the magnitudes of the ohmic resistances r which should be placed in a column adjacent to that containing the apparent resistances R. The ohmic resistance of the condenser itself is very large and is not to be included. The values of the reactances and the out-of-phase components of the sections must be given the proper + or − sign to designate their inductive or capacitive character.

Include also the sum of the three magnitudes of P, of R, and of $\cos\theta$ and the algebraic sum of the magnitudes of X and of $E\sin\theta$. These sums should equal the magnitudes of the respective quantities determined experimentally for the three sections in series. The lack of agreement in the values indicates, in a measure, the magnitude of the actual errors.

11. Vector Diagrams.—Represent by means of the two forms of the vector diagram (Fig. 30) the e.m.fs. E_1, E_2, and E_3 impressed, respectively, on the sections R_0, L_0, and C_0.

The positions of the vectors are determined by means of the tabulated components $E\cos\theta$ and $E\sin\theta$. The in-phase or RI-drop components all contribute to the resultant in-phase component in the same direction, while out-of-phase components in the inductive and capacitive sections oppose each other. The e.m.f. E_1 impressed on the "noninductive" section is shown as having a small inductive reactance, as is the case when an ordinary rheostat forms this section.

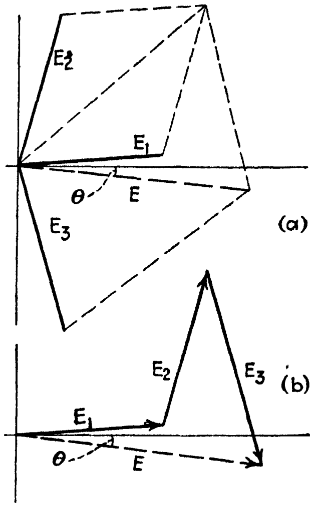


FIG. 30.

In the diagram (a) the e.m.fs. are represented as diverging from a common point, and in the diagram (b) one vector begins where the other ends. The impressed e.m.f. on the three sections together is the resultant E of the three vectors determined in each diagram as represented. Note that the diagram (b) is equivalent to diagram (a) and, when understood, is simpler.

Determine E and θ from the diagrams, and compare them with the tabulated values.

EXPERIMENT 19

DENSITY OF MAGNETIC FLUX. COMPARISON METHOD

Apparatus.—Jerk-type exploring coil; electromagnet with shelf for supporting the exploring coil in its starting position; resistance box; ballistic galvanometer with short-circuiting key (see Chap.

VII, Exp. 30); Hibbert or Cenco magnetic standard (see Chap. VII, Exp. 47); ammeter (5-amp range); rheostat; battery.

Method and Manipulation.—Connect the two circuits as shown in Fig. 31(*a*),(*b*). The resistance in *R* is adjusted until the galvanometer is critically damped; *i.e.*, in returning to the null point after a throw, it just returns to the null point without crossing it. Since the null reading changes (Exp. 15) on reversing the direction of deflections or throws, the throws must all be taken in the same direction. For this and other reasons the

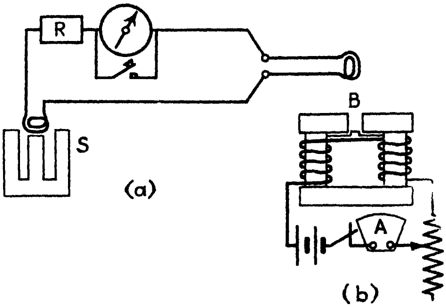


FIG. 31.

galvanometer is short-circuited both while raising the coil of the magnetic standard and while placing the exploring coil *E* into the magnetic field *B* to be measured.

With the exploring coil in the magnetic field, drop the coil of the standard through the known magnetic field, observe the throw, and then jerk the exploring coil from the magnetic field. The throws from both, as already explained, should be in the same direction (left to right). The first throw is discarded to establish constancy of the null point, and the galvanometer scale is adjusted to read zero. At least three sets of readings are taken. Then from

$$e = \frac{N\phi}{10^8 t} = Ri, \qquad N\phi = 10^8 QR,$$

whence

$$\frac{N_x\phi_x}{N_s\phi_s} = \frac{10^8 Q_x R}{10^8 Q_s R} = \frac{Q_x}{Q_s} = \frac{d_x}{d_s},$$

from which

$$\phi_x = \frac{N_s \phi_s}{N_x} \frac{d_x}{d_s} \text{ maxwells,} \tag{1}$$

and

$$\beta = \frac{\phi_x}{A} \text{ gauss.} \tag{2}$$

EXPERIMENT 20

CAPACITANCE. COMPARISON METHOD

Apparatus.—Paper condenser whose capacitance is to be measured; standard mica condenser; ballistic galvanometer; condenser discharge key; battery; protecting resistance of about 10,000 ohms, which is to be placed in series with the battery.

Method and Manipulation.—Connect apparatus as shown in Fig. 32. The discharge key K has three springs which can be depressed in rapid succession by one tap. This action disconnects the charging battery, discharges the condenser through the galvanometer, and then disconnects the condenser. These three operations occur within about 0.01 seconds; but this interval is long enough for the condenser to discharge all the measurable part of its free charge. This free charge is always considered to be the measure of the capacitance of the condenser. The bound "absorbed charge" due to the displacement of the comparatively few free electrons in the imperfect dielectric is usually small and is liberated slowly. The condenser is disconnected by the key quickly in order to prevent any appreciable part of this absorbed charge, which may be liberated during the period of the throw, to affect the galvanometer. The key must be held in the depressed position until the throw reading is taken.

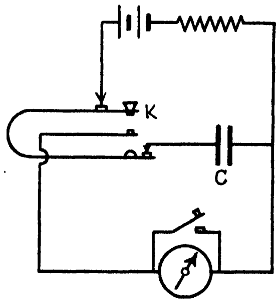


Fig. 32.

Three consistent throws are taken from the discharge of the paper condenser C_x whose capacitance is to be measured, and then three from the standard mica condenser C_s. Then, since the

charging e.m.f. is the same for both condensers,

$$C_xE = Q_x;$$
$$C_sE = Q_s.$$

Then

$$\frac{C_xE}{C_sE} = \frac{Q_x}{Q_s} = \frac{d_x}{d_s}:$$

from which

$$C_x = \frac{d_x}{d_s} C_s \text{ farads.}$$

The first observed throw is discarded, and the zero point is reset. All throws are taken to the right; and, while the coil is returning to the null point, the galvanometer is short-circuited at intervals to bring the coil to the null point without crossing it. If it is difficult to bring the coil to rest at zero, discharge the condenser when the coil is at its right-hand elongation, but measure throws from the open-circuit zero. (See more complete discussion in Exp. 16 and in Chap. VII, Exp. 31.)

QUALITATIVE EXPERIMENTS

These qualitative experiments are extra experiments to be performed by each student individually in addition to the assigned quantitative experiments. They are to be performed at the proper time to supplement the subject matter in the text part of the course in electricity. It is intended that the apparatus be connected and adjusted by the instructor and that the student need not spend more than a few minutes on each experiment nor be required to make any report.

A. ATTRACTION AND REPULSION BETWEEN ELECTRIC CHARGES (B.P.1.)

Charge one gilded pith ball with + electricity and the other with −. Note that a + charge attracts one of the charged pith balls and repels the other and that a − charge does the reverse. Formulate B.P.1. Also note that a neutral pith ball is attracted by either of the two kinds of charges.

B. MISCELLANEOUS EXPERIMENTS IN ELECTROSTATICS

Apparatus.—Electroscope; two hard-rubber rods, one of which has a woolen cloth attached to one end; glass rod in insulating handle; woolen cloth; silk cloth; metal beaker; proof plane.

Precaution.—Care must be taken not to tear the leaves of the electroscope by bringing an excessive charge too close to the plate or by bringing any charge to the side of the electroscope. Charge the rod or proof plane at a distance, and then bring it slowly downward toward the plate of the electroscope, and stop when the leaves diverge a reasonable amount.

In all explanations use the concepts of the electric field and potential.

1. Charging Electroscope by Contact and by Induction and Determining the Sign of Charge.—*a.* Collect on a proof plane a negative charge from the hard-rubber rod which had been charged by rubbing with wool. Then touch the plate of the electroscope with the charged plane. Note that on bringing the negatively charged rod to the now negatively charged electroscope the leaves diverge more and that a positively charged glass rod converges the leaves. Explain.

b. Bring a negatively charged rod into the neighborhood of the plate of an uncharged electroscope without touching it. Then touch with the hand (or ground) the plate of the electroscope, and remove the charged rod. Explain why the electroscope has a positive charge, and check your conclusion by testing as in *a.* Repeat by charging the electroscope by means of a positively charged glass rod.

c. Note the effect of bringing the hand or a grounded sheet of metal near to the plate of the charged electroscope. Explain why a divergence rather than a convergence of the leaves is the test for the presence of a charge on any body.

d. Determine the sign of the charge produced on hard rubber after it has been rubbed with silk by means of the divergence produced in a charged electroscope. Determine also the sign of the charge on glass after it has been rubbed with wool.

2. Faraday's Ice-pail Experiment.—Place a metal beaker on the plate of the electroscope, and bring a charged proof plane into the beaker without touching it. Then ground the electroscope by touching the beaker or plate, and after removing the ground remove the proof plane. The electroscope now has a charge opposite in sign to that on the proof plane. Now touch the beaker with the proof plane, and show that the induced and inducing charges are equal and opposite in sign.

What purpose is served by the beaker?

3. Production of Equal Opposite Charges by Friction (Contact).—Place the metal beaker on the plate of the electroscope. Hold one end of each of the two hard-rubber rods in succession within the beaker, using the cloth-covered end of one of them. If either rod is shown to be charged, remove the charge by passing the rod rapidly through a bunsen flame or by holding it in the ionized air near an electric arc. Then, holding the ends of the rods within the breaker, rub the woolen cloth of one against the hard rubber of the other. Show that the charges produced by the friction are equal and opposite in sign.

Why is it necessary to use the beaker in this experiment?

C. Oersteds Experiment (B.P.2)

A wire which is a part of an electric circuit is placed above and in line with the horizontal axis of an undeflected magnetic needle. Observe that when the circuit is closed, the magnetic needle deflects, and that when the direction of the current is reversed, that of the deflection also reverses. Observe also the interaction between wires carrying electric currents, and demonstrate B.P.2.

D. Magnetic Loop

A coil of aluminum wire is suspended in the earth's magnetic field from mercury cups and is carefully adjusted. When a large current energizes the coil, the coil turns until one of its planes faces north. On reversing the current, the coil turns until its other side faces north.

Explain in terms of B.P.2. why the coil turns and, also, how the energized coil represents an elemental magnet or any magnet.

E. Magnetic Lines of Force about an Energized Wire (Laws A_1A_2)

One thick wire, which is part of an electric circuit, or one side of a coil of several turns is passed through a hole in a horizontal glass plate.

1. Spread iron filings thinly about the wire, and then close the electric circuit. Note that on tapping the glass plate the iron filings tend to form into concentric circles about the wire (law A_1).

2. Open the circuit. Then place two weak magnets on either side of the wire with their unlike poles facing each other. Spread iron filings over the glass, and, tapping, note the direction of the magnetic lines of force through and about the wire.

3. Then close the electric circuit, and, tapping, note the resultant of the two superposed fields (law A_2, electromagnetic reaction).

4. Reverse current, and note the change in the lines. Note on which side the magnetic field is strengthened and therefore in which direction the wire is urged.

5. Number 1 may be tried with small compasses in place of iron filings. Then note the change in the direction of the magnetic lines on reversing the current.

The different parts of this experiment may be shown with separate pieces of apparatus, each adjusted for its special purpose.

F. Magnetic Field about Two Bar Magnets

Two bar magnets are placed at right angles to each other on a large sheet of paper. The magnetic field about the magnets then is the resultant of the earth's magnetic field and the two fields of the magnets superposed. Lines which have been drawn about the magnets by the instructor show the directions of the magnetic lines of force in a horizontal section of the field. The student moves a small compass along each of several such lines and notes that the needle of the compass always points in the direction of the line on which the compass is resting.

G. Electromagnetic Reaction

A long flexible conductor is suspended so that a part of it passes through a strong magnetic field between the poles of an electromagnet. Observe the reaction (law A_2) when the current energizes the wire. Reverse the current.

H. Capacitance

Place a grounded sheet of metal directly above the plate of a charged electroscope without touching the plate. Observe and explain the convergence of the leaves.

Test a thick flat piece of hard rubber for electrification, and remove the charge (see Exp. B, 3) if there is any. Then insert the hard-rubber dielectric between the grounded plate and the

plate of the electroscope. Explain the further convergence of the leaves.

Explain how these two observations illustrate the action of the electric condenser.

I. ELECTROLYSIS

Observe the decomposition of water by electrolysis and the deposition of a metal.

J. VOLTAIC CELL

Connect a copper and zinc plate, which are held in fixed position by a supporting frame, to the leads of a portable galvanometer. Then immerse the plates in a tumbler of dilute H_2SO_4. Observe the current and the polarization.

K. E.M.F. PRODUCED BY RELATIVE MOTION BETWEEN A CONDUCTOR AND A MAGNETIC FIELD (B.P.3)

Connect a coil of wire to a galvanometer and move it through the magnetic field of a bar magnet. Observe the galvanometer deflection due to the current which is produced by the e.m.f. generated in the coil by its motion. Explain in terms of law *A* and B.P.2.

Now, with the coil at rest, move the magnetic field through the coil by moving the magnet. Observe the induced e.m.f., show that it cannot be explained in terms of B.P.1 or B.P.2, and explain by B.P.3.

Repeat the experiment using an energized solenoid in place of the bar magnet.

L. THERMOCOUPLE

Place a thermocouple in circuit with a galvanometer, and adjust the resistance of the circuit to give critical damping. Observe the galvanometer deflections first when one junction of the couple is heated and then when the other is. The temperature of the fingers is sufficient when a sensitive galvanometer is used.

M. PHOTOVOLTAIC CELL

Place a photovoltaic cell in series with a critically damped galvanometer, and observe the variation in the e.m.f. generated in the cell by different intensities of illuminations.

N. Body E.M.F.

Connect two long copper-wire leads to a sensitive moving-coil galvanometer which is critically damped. Hold the ends of the connecting leads, one in each hand, between two fingers, which may be moistened if necessary. Observe that a potential difference exists between the two hands.

O. Mutual Induction

Connect one coil, the primary, into circuit with a battery, rheostat, and ammeter; and place another coil, the secondary, in circuit with a critically damped galvanometer. Place the coils with their faces parallel to each other, and observe the throws in the galvanometer whenever the current in the primary coil is altered either by changing the resistance in the rheostat or by opening or closing the circuit. Explain the observed action by means of electromagnetic pulses and their motional electric fields (B.P.3) (law *C*).

Place the two coils with their faces at right angles to each other, and observe whether similar throws are obtained when the current in the primary circuit is altered. Explain.

P. Self-induction

Place the solenoid of an electromagnet or one of the coils of a transformer into the *X* arm of a Wheatstone box bridge. Observe that, when the bridge is properly balanced and the galvanometer circuit closed, a closing or opening of the battery circuit produces a throw on the galvanometer. The inductive resistance in the *X* arm contains a counter e.m.f. of self-induction during the interval when the current is changing within the coil. This counter e.m.f. produces a current in the bridge network in the same manner that any e.m.f. would if it were introduced into that arm for that instant. Note, also, that if the battery circuit is closed before that of the galvanometer, the throws do not appear.

Q. Transformer

Place an alternating-current ammeter into the primary circuit of a transformer, and observe that the magnitude of the current in the primary circuit depends on whether the secondary circuit is open or closed.

R. THREE-ELECTRODE VACUUM TUBE

Connect the three-electrode vacuum tube as shown in Fig. 33.

1. With the battery-C circuit open, note that when the $-$ end of the B battery is connected to the plate P there is no plate current and that when the $+$ end is connected to the plate the magnitude of the plate current depends on the temperature of the filament F, which is varied by adjusting the resistance R.

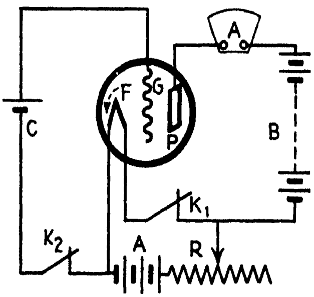


FIG. 33.

2. Observe the change in the plate current when the key K_2 is closed, and how the change depends on whether the $+$ or $-$ end of the battery C is connected to the grid G.

S. PHOTOELECTRIC CELL

Connect a photoelectric cell in circuit with a battery and a galvanometer, attaching the negative side of the battery to the photoactive material of the cell. Note that the current depends on the quantity of light falling on the photoactive material.

T. CONSTRUCTION OF ELECTRIC INSTRUMENTS OBSERVED

At appropriate times in the course observe the internal construction of resistance box, galvanometer, ammeter, voltmeter, wattmeter, etc., under the direction of the instructor.

CHAPTER VII

ELECTRICITY II

This chapter is designed to be the laboratory part of a one-quarter, three-credit course in electricity measurements and to be taken after the completion of a general course in college physics. This course requires two 2-hour periods per week in the laboratory when a 1-hour supplemental lecture is a part of the course.

EXPERIMENT 21

ADJUSTMENT OF MOVING-COIL GALVANOMETER

Reread the description of the moving-coil galvanometer given in Chap. VI (Intr. 1), and perform all the operations and adjustments listed in Chap. VI (Exp. 16). Use the 3-mil upper suspension which is standard for most purposes where ordinary sensitiveness is sufficient. Greater sensitivity than is required unnecessarily adds to obvious difficulties and increases the zero shift. Decrease the zero shift by making the axis of the coil coincide as nearly as is practical with the axis of the iron cylinder. The damping rectangle should be on the coil before it is raised.

The suspensions are of gold or phosphor-bronze strip. The usual resistance of the coil with the suspensions is about 130 ohms. The temperature coefficient of the galvanometer depends on the temperature coefficients of the magnetic field, the torque of the suspensions, and the expansion of the material of the coil, and on whether the galvanometer is used for current or for ballistic measurements. When the suspensions are of phosphor bronze, the Leeds and Northrup P-type galvanometer has a temperature coefficient of $+0.00018$ for current deflections and -0.00017 for ballistic throws.

In a perfectly adjusted galvanometer having a perfect radial field, the current is proportional to the angle of deflection; and if the scale is a perfect circle with its center of curvature in the axis of the coil, the current is proportional to the observed deflection. These conditions are never fully realized. For precision work, it

is therefore necessary to correct the observed throws by means of a correction curve.

In a perfect galvanometer, the torque due to electromagnetic reaction is

$$L = BlNDI' = T\theta.$$

Then

$$I = 10I' = \frac{10T}{BlND}\theta = K_1\theta = Kd \text{ amp}, \tag{1}$$

where d = the deflection on a circular scale, and K = a constant whose magnitude is the current per 1-cm deflection.

Questions.

1. How is the radial field of a galvanometer produced?
2. What causes the zero shift after a first deflection?
3. What adjustment of the galvanometer decreases or entirely eliminates the zero shift?
4. Explain why the damping rectangle checks the oscillations of the coil?
5. Why must the coil be lowered before shifting the galvanometer even slightly?

EXPERIMENT 22

SENSITIVITY OF A MOVING-COIL GALVANOMETER

Apparatus.—One storage cell; three resistance boxes R, S, T, of which T may be one or any collection of known resistances whose total resistance is about 20,000 ohms; rheostat; voltmeter; switch; key; damping coil and magnet M; the galvanometer.

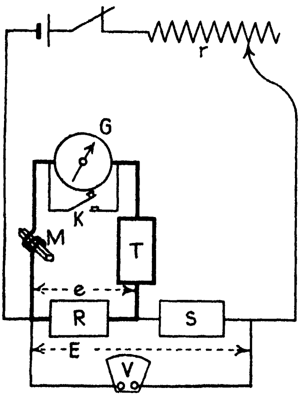


FIG. 34.

Method and Manipulation.—Connect the apparatus as shown in Fig. 34. Use the galvanometer with 3-mil upper suspension and a damping rectangle. If the rectangle alone produces critical damping, the short-circuiting key K and the damping coil and magnet M need not be inserted. In any case, the coil should never be allowed to cross the zero line to the apparent left by more than a few millimeters in order to avoid a zero shift. In case the coil is underdamped, the magnet is

moved within the coil M to generate a current in the proper direction to prevent the swing of the coil from moving beyond the final deflected position or beyond the null point. The scale zero mark is placed directly above the axis of the telescope, and the null reading is adjusted to the zero mark, as nearly as is practical, by turning the lug which supports the upper suspension.

Make $R = 10$ ohms and $T + G + M = 20{,}000$ ohms. Then only 0.05 per cent of the current in R is diverted to the galvanometer, and the current in R and therefore the potential difference across R may be practically unaffected by diverting so small a part of the whole current. The resistance in S is adjusted until the galvanometer deflection is about 10 cm. The magnitude of the resistance in S may then be several hundred ohms. If the e.m.f. of the storage cell does not exceed the range of the voltmeter, the rheostat r may be discarded.

Take a preliminary deflection of the magnitude to introduce the proper zero shift which is due to magnetic impurities in the coil and imperfections in the galvanometer adjustment and in the uniformity of the radial field. Then, discarding this first reading, again adjust the scale to read zero at the null point, and take the final galvanometer and voltmeter readings. Repeat three times, and use the average values. If the galvanometer zero still shifts, the null reading taken immediately after a deflection is used in calculating the deflection. The null reading of the voltmeter must also be observed and recorded.

The potential difference e across R is determined from

$$\frac{e}{E} = \frac{R}{R + S},$$

whence

$$e = \frac{R}{R + S} E \text{ volts.}$$

Then the current energizing the galvanometer is

$$I = \frac{e}{T + G + M} \text{ amp.}$$

Assuming proportionality of the current to the deflections,

$$I = fd \text{ amp,}$$

from which the constant or current sensitivity

$$f = \frac{I}{d} \text{ amp/cm.} \tag{1}$$

The sensitivity of a galvanometer is often rated commercially in terms of the current in amperes per millimeter. Then

$$\text{Current sensitivity } f_1 = \frac{I}{10d} \text{ amp/mm} \tag{2}$$

where d represents deflection in cm.

The sensitivity is also designated in terms of the number of megohms through which 1 volt causes a deflection of 1 mm. Then

$$\text{Megohm sensitivity} = \frac{1}{10^6 f_1} \text{ megohms.} \tag{3}$$

In purchasing an instrument the sensitivities are listed in terms of a scale placed at a distance of 1 m from the galvanometer. Then

$$\text{Commercial current sensitivity} = f_1 \cdot \frac{D}{100} \text{ amp/mm} \tag{4}$$

where D = the distance of the scale in cm used with the galvanometer when the sensitivity f_1 is determined.

$$\text{Commercial megohm sensitivity} = \frac{1}{10^6 f_1} \cdot \frac{100}{D} \text{ megohms.} \tag{5}$$

Calculate the current sensitivity f and the commercial current and megohm sensitivities of the galvanometer.

Questions.

1. Why is the potential difference across R practically unchanged by diverting the current I through the galvanometer? How could a correction be made for the slight change?
2. Why must the voltmeter be attached across $R + S$ at the time when the galvanometer deflection is being read?

EXPERIMENT 23

ELECTROMOTIVE FORCE OF VOLTAIC CELL. POGGENDORFF'S METHOD

Apparatus.—The voltaic cell whose e.m.f. E_x is to be measured; battery of two storage cells; resistance box; low-range milliammeter; rheostat; slide-wire resistance; high resistance HR; galvanometer.

Method and Manipulation.—Connect the apparatus as shown in Fig. 35, being careful to have the potential difference across R

in the main circuit oppose the e.m.f. being measured in the secondary circuit.

1. The proper magnitude of the resistance R to be used is calculated from the estimated magnitude of E_x and a nearly full-scale reading of the milliammeter by means of the equation

$$R = \frac{E_x}{I} \text{ ohms.}$$

The current used, however, must not exceed the carrying capacity (0.25 watts) in any individual coil used in the resistance box R.

2. The resistance R is now inserted in box R, and the rheostat r is adjusted to give approximately the current reading employed in the calculation of R. The potential difference e across R is then approximately equal to the e.m.f. of E_x to be measured. The rheostat r and the slide wire S are adjusted until depressing the key K produces no deflection in the galvanometer. The potential difference e across R tends to produce a current in the E_x circuit, but it is balanced by an equal and oppositely directed potential difference of the cell E_x. This potential difference, impressed across R by E_x, is a measure of the e.m.f. of E_x when no current is flowing through the cell.

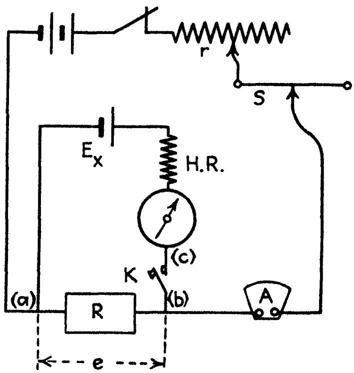


Fig. 35

That $e = E_x$ when there is no current in the galvanometer can be seen clearly from the following analysis: Let key K be open. Then the potential of the $-$ plate of E_x must be that of the point a in the main circuit. The potential at c must be that of the $+$ plate of E_x. Then if $e = E_x$, the potential at b is that at c, and consequently closing the secondary circuit by the key K can produce no electron flow.

The object of the high resistance HR is to protect the galvanometer and the cell against too large a current in the initial attempts to obtain a balance. The high resistance used usually has a magnitude of about 20,000 ohms but may be varied either to

increase or to decrease the sensitiveness of the balance. The sensitiveness of the adjustment must be at least such that a change of one small division on the milliammeter produces a deflection of one small division on the galvanometer. Test the sensitivity and the plug contacts in R before taking the final readings. Repeat with at least one slightly different resistance in R. Then

$$E_x = e = RI \text{ volts.} \tag{1}$$

Tabulate the observations and results; and indicate the probable error, treating the constant probable adjustment error of 0.1 per cent in the resistance R as though it were a probable error.

The e.m.f. of standard cells was obtained by this method, except that the current was measured more accurately by means of an electrodynamometer or current balance.

Questions.

1. Why must the e.m.f. of the main battery be larger than that of the cell E_x?
2. Why must the e.m.f. of E_x oppose the potential difference across R?
3. How is it known that the milliammeter measures the current in R?
4. What is the optimum magnitude of the current?
5. How is the proper magnitude of the resistance to be placed in R calculated?
6. What adjustment is made before testing for balance in order that the e.m.f. of the cell may be approximately equal to the potential difference across R? Why should this adjustment be made?
7. Why is the slide-wire resistance used?
8. What is done to test whether the galvanometer is sufficiently sensitive?
9. Does the high resistance HR alter the balance? What does it alter?
10. Why is it not desirable to have the galvanometer more sensitive than is necessary?
11. If a thermoelectric e.m.f. exists in the secondary circuit, how does it affect the result? How can the error due to it be eliminated? How should you test for the presence of such an e.m.f.? Is the sensitivity of the galvanometer as used in the experiment high enough to expect any appreciable effect?

EXPERIMENT 24

ELECTROMOTIVE FORCE OF VOLTAIC CELL POTENTIOMETER METHOD

Apparatus.—Standard cadmium cell; dry cell whose e.m.f. is to be measured; etc. The cadmium cell is used as a standard with

which the magnitude of other e.m.fs. is determined by comparison. The e.m.f. of the cell is 1.01830 volts at 20°C., but owing to slight impurities the e.m.f. of individual cells may differ from that value by several points in the fourth decimal place. The temperature coefficient is negligible at ordinary room temperatures when the required accuracy of measurement does not exceed 0.1 per cent (see Appendix 32).

The cell polarizes so that only as minute a current for the shortest possible length of time is allowed to be taken from it as is necessary in the process of balancing the potentiometer. The

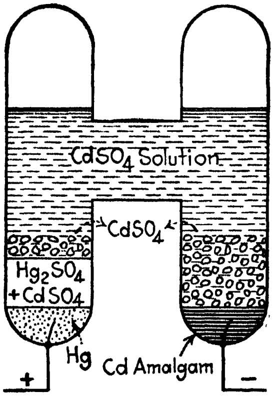


Fig. 36.

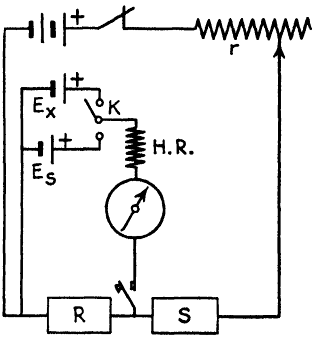


Fig. 37.

high resistance placed next to the cell aids in protecting it. The cell is not used for the production of a current; it must not be connected to a voltmeter; and not even should the two binding posts of the cell be touched at the same time. The construction of the cell is shown in Fig. 36.

The e.m.f. of the cadmium cell was originally determined by Poggendorff's method in terms of a current and a known resistance as defined by an international conference (see Appendix 31).

Method.—The potentiometer is an arrangement of circuits for comparing potential differences by a modification of Poggendorff's method. The arrangement of the circuits for measuring the e.m.f. of a voltaic cell E_x by comparison with that of the standard cell E_s is shown in Fig. 37. The e.m.f. of the battery in the main circuit must be higher than that which is to be measured.

The current in the main circuit, although another value could be used, is adjusted here to a magnitude of 0.001 amp. The potential difference along the circuit then is 1 volt per 1000 ohms. In order that the potential difference across R may, with such a current, be equal to the e.m.f. of the standard cell, the resistance in the box R must be, to within the nearest ohm,

$$R = 1000E_s.$$

The approximate resistance $S + r$ can be calculated from the resistance R and the total resistance in the circuit required for the battery to produce the current of 0.001 amp. The greater part of this calculated resistance is placed in S and, for convenience, the part is such that the resistance $R + S$ is an exact multiple of 100 ohms. The remainder of the calculated resistance is placed in r. The potential difference across R, then, before any test for balance is attempted, is expected to be nearly equal to E_s. This calculation and adjustment save time, protect the cell, and are the accepted practice.

The experimental test for balance is now made, and the balance is completed by adjusting r. The sensitivity of the galvanometer is tested by noting the change in deflection produced by changing the resistance in S by 1 ohm from its magnitude at balance. If this test shows that the galvanometer is sufficiently sensitive to detect easily a change of 0.1 per cent in the total resistance of the circuit, the adjustment of the current is correct to within the desired 0.1 per cent. If either R or S is a plug box, a test must also be made for poor plug contacts before accepting the adjustment for balance.

The standard cell now has served its purpose in the adjustment of the current in the main circuit to a value

$$i = 0.001000 \pm 1 \text{ amp.}$$

The standard cell is then replaced, by means of the switch K, by the cell E_x whose e.m.f. is to be measured. The approximate value of the e.m.f. of the cell is usually known, so that the new resistance R_x to be placed in the box R for approximate balance can be calculated. Since the current in the circuit must not be altered, any change of resistance of R must be compensated by an equal opposite change in S.

The final balance is made in the same manner as in the foregoing experiment, except that the total resistance in the circuit must not be changed. The balance is obtained therefore by interchanging resistances between the boxes R and S, being careful to keep the sum of the two resistances unaltered. Then

$$E_x = e_x = R_x i = 0.001\ R_x \text{ volts.} \tag{1}$$

It should be observed how, through the foregoing operation, this method compares the e.m.f. of one cell with that of the other and that any potential difference can be measured by the same method.

The working standard cell should at times be compared with another standard cell which is reserved for such comparisons.

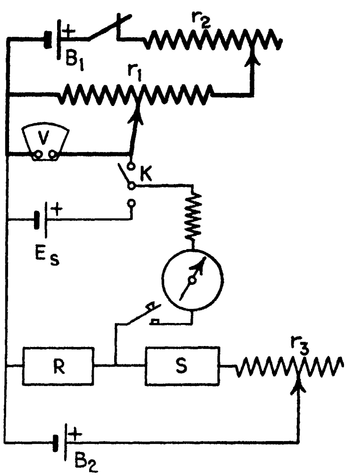


FIG. 38.

EXPERIMENT 25

CALIBRATION OF VOLTMETER. STANDARD POTENTIOMETER METHOD

Apparatus.—The voltmeter to be calibrated, which has a range of 1.5 volts; other potentiometer equipment, as shown in Fig. 38.

Guard against magnetic effects of rheostats.

Method and Manipulation.—The cell B_1 is connected as shown in series with the high-resistance rheostat r_1 and the low-resistance rheostat r_2. The voltmeter to be calibrated is attached so that it can easily be made to span any desired portion of the rheostat r_1. In this manner the potential difference across the binding posts of the voltmeter can be varied at will and then accurately adjusted by means of r_2. The remainder of the apparatus is a potentiometer arranged for measuring the potential difference across the binding posts of the voltmeter by comparison with the standard cell. The rheostat r_3 may be replaced by a resistance box having 0.1-ohm coils.

The potential difference is measured by the potentiometer for voltmeter readings at or near each of the marked divisions on

the scale. In each case the proper interchange between R and S is made to insure approximate balance before testing. The final balance is obtained by continuing the interchange between R and S or by altering the potential difference across the voltmeter by changing r_2. The potentiometer and voltmeter should in each case give the same reading. Any observed difference gives the correction which when added to or subtracted from the observed voltmeter reading gives the correct potential difference across the binding posts of the voltmeter. If the null reading of the voltmeter is not zero, the correction for the null reading is made before the voltmeter reading is compared with that of the potentiometer.

Since the current in the main potentiometer circuit may change with time, the standard cell balance is checked after every two or three points have been tested. If a change has taken place, the circuit is rebalanced.

When, for example, the voltmeter reads 1.200 volts, the trial resistance placed in the spanned box is

$$R_T = 1000\ E = 1200 \text{ ohms.}$$

If, for final balance, this resistance must be changed to 1206, the actual potential difference across the binding posts of the voltmeter is

$$E_x = 0.001\ R = 1.206 \pm 1 \text{ volts.} \tag{1}$$

The correction for the voltmeter reading at the point 1.2 then is $+0.006 \pm 1$ volts.

Tabulate the following observations and derivations for each tested point: room temperature, null reading of voltmeter, observed voltmeter reading, voltmeter reading (corrected for null displacement), correct voltage, correction to be applied to voltmeter reading.

Draw the correction curve for the voltmeter from the tabulated data, using the conventions given in Chap. VI (Exp. 1). Whenever a voltmeter reading is taken, hereafter, a correction is to be made first for the zero displacement and then for the correction given by the correction curve. Additional data for this curve will be obtained in Exp. 26.

If only one Edison storage cell is available for each of the main circuits, the voltmeter can be calibrated by this method only to about 1.3 volts.

Care must be taken when using ordinary rheostats, because their contacts often are poor, and if the rheostats are magnetic and too close to the voltmeter they affect the readings. If tapping the voltmeter lightly changes the voltmeter reading, repeat the tapping at each observed point. The voltmeter leads must have a negligible resistance compared to that of the voltmeter. The resistance of No. 20 copper wire is 0.010 ohms per foot; and of No. 24 wire, 0.025 ohms.

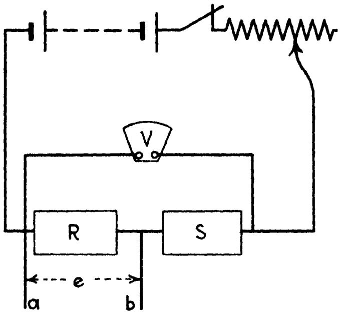


Fig. 39.

The student should bear in mind that in calibrating a voltmeter he is measuring potential differences, and that any potential difference or e.m.f. in a circuit can be measured by means of the potentiometer.

The temperature coefficient depends greatly on the range of the voltmeter; for example, for a

1.5-volt range,	$\alpha =$	$+0.00013$ per °F.
15.-volt range,	$\alpha =$	-0.00005 per °F.
150.-volt range,	$\alpha =$	-0.00007 per °F.

Higher range voltmeters may be calibrated by any one of the following four methods:

1. The *volt-box method* employs two resistance boxes as shown in Fig. 39. These boxes, for high voltages, may have to be special boxes whose carrying capacities are larger than 0.25 watts. The potential difference e across R is measured by a potentiometer, not shown, through the connectors a and b. Since the same current energizes both R and S, the potential difference across $R + S$ is

$$E = \frac{(R + S)e}{R} \text{ volts.} \tag{2}$$

The difference between this and the voltmeter reading gives the correction for that reading.

2. The *current-resistance method* employs a known resistance in series with the voltmeter whose resistance R_V also must be known or measured (see Fig. 40).

The potential difference e across R is measured by means of a potentiometer. Then the current in both R and the voltmeter is

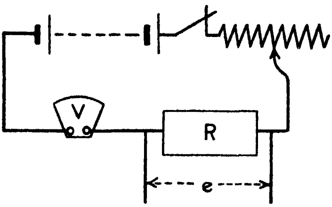


FIG. 40.

$$i = \frac{e}{R} \text{ amp.} \qquad (3)$$

The potential difference across the binding posts of the voltmeter is

$$E = R_V i \text{ volts.} \qquad (4)$$

3. If the high-range voltmeter has also a 1.5-volt scale, the correction curve for the low range is drawn from data obtained by the standard potentiometer method and the resistances of the voltmeter for the two ranges are measured. Since the same current, regardless of the range used, produces the same deflection on the voltmeter, the current at any given deflection is

$$i = \frac{e}{r} = \frac{E}{R} \text{ amp,}$$

from which the correct voltage on the high-range scale is

$$E = \frac{Re}{r} \text{ volts,} \qquad (5)$$

where e is the correct voltage for that deflection on the low-range scale.

Any voltmeter can be made into one of higher range by adding an appropriate external resistance, called *multiplier*. A 150-volt range, for example, is made from one of 1.5-volt range by adding a resistance x which makes the total resistance R of the voltmeter one hundred times that of the lower range instrument. Then

$$R = 100r = x + r,$$

and

$$x = 99r \text{ ohms.} \qquad (6)$$

4. A voltmeter may also be calibrated by comparing its readings with a standardized voltmeter. Both instruments must be

connected at the same time across the same resistance when the comparison readings are being taken.

Alternating-current voltmeters may be calibrated as follows:

1. Thermocouple and hot-wire alternating-current voltmeters give the same readings with either alternating-current or direct-current voltages. Such instruments therefore are standardized as direct-current voltmeters.

2. All other types of alternating-current voltmeters can be calibrated by comparison with thermocouple or hot-wire voltmeters which have been standardized by the foregoing direct-current method.

3. The dynamometer-type voltmeter can be calibrated as a direct-current voltmeter, except that a correction must be made for reactance. The resistance and self-inductance of the instrument therefore must be measured.

EXPERIMENT 26

CALIBRATION OF VOLTMETER. MODIFIED POTENTIOMETER METHOD

Apparatus.—The voltmeter standardized in Exp. 25 and accessories as shown in Fig. 41.

Method and Manipulation.—Figure 41 shows that in this modification the same battery energizes both the main potentiometer circuit and the voltmeter and that although the current in R is that in S, the current in r is the sum of those in the two branches of the divided circuit.

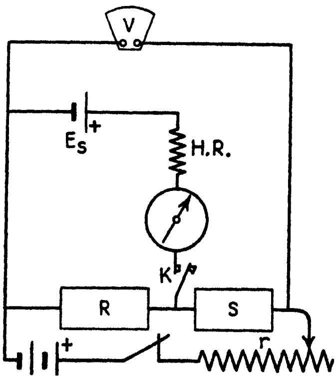


Fig. 41.

If the resistance placed in R is equal to $1000E_s$, and the E_s circuit is balanced, the potential difference across $R + S$, which the voltmeter is supposed to read, is

$$E_x = 0.001(R + S) \text{ volts.} \quad (1)$$

Then, for example, if it is desired to check the voltmeter reading when the potential difference across its binding posts, *i.e.*, across $R + S$, is 1.5 volts, when the e.m.f. of the standard cell is 1.018 ± 1

volt, the resistance placed in the two boxes is

$$R + S = 1018 + 482 = 1500 \text{ ohms.}$$

Before testing for balance, however, r is adjusted until the voltmeter reads the to-be-established potential difference across its binding posts. The potential difference across R then is approximately equal to E_s; and the apparatus is ready for the final adjustment of the potentiometer balance, which is made by altering r.

If at final balance the voltmeter reading (corrected for zero displacement) is 1.496 volts, and the potential difference impressed across the binding posts of the voltmeter is 1.500 volts, the correction at the point 1.496 (not 1.500) is +0.004 volts.

Find the corrections for the voltages represented by the large scale divisions marked higher than the e.m.f. of the standard cell, and tabulate the observations and the corrections. Add these corrections to the plot of the correction curve of Exp. 25, and complete the curve with revision if necessary. Variations as high as 0.2 of the smallest scale division may be expected.

Questions.

1. Is the potential difference across $R + S$ the same regardless of whether or not the voltmeter circuit is opened or closed?

2. What is the approximate resistance of an ordinary voltmeter per 1-volt range?

3. Why must the resistance of a voltmeter be large compared with the resistance across which it measures the potential difference? Why is this condition not required while the voltmeter is being calibrated? Is the voltmeter designed to measure the potential difference between its binding posts regardless of the resistance across which the potential difference is being measured? Do the connecting wires introduce an appreciable error?

4. What is the function of the standard cell in the potentiometer?

5. Is the current in S the same as in R? As in r?

6. How is the proper magnitude of R determined? Of S? Of r?

7. Will the balance change if the voltmeter circuit is opened? Will the total current? Will the current in $R + S$?

8. Why is it unnecessary to have the resistance in R nearer to $1000E_S$ than to within the nearest ohm? Why is resistance in S adjusted only to within the nearest ohm?

9. Is an appreciable error introduced if the $-$ pole of E_S is connected to the $-$ post of the voltmeter in place of that of the resistance box? What is the resistance per meter of the connecting wire used in the laboratory?

EXPERIMENT 27

CALIBRATION OF AMMETER. STANDARD POTENTIOMETER METHOD

Apparatus.—A milliammeter (150 ma range) which is to be calibrated; a low-resistance standard T (Fig. 42) made to carry the maximum current to be used and of such resistance that the maximum potential difference may be large but not larger than can be measured directly with the potentiometer. This limit is usually 1.5 volts, but with the apparatus here illustrated it is 1.2 volts because of the lack of sufficient e.m.f. in one Edison storage cell. A special 8-ohm coil is conveniently used for T, although an ordinary resistance box could be employed with such small currents. Slide-wire resistance r_2 and other apparatus, as shown in Fig. 42.

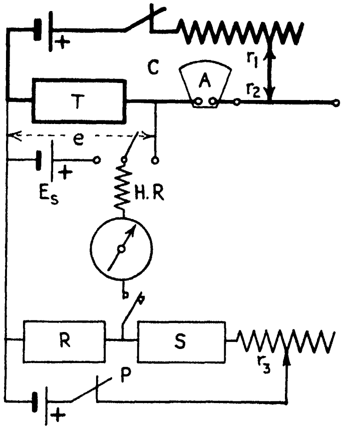


Fig. 42.

Method and Manipulation.—The apparatus consists of two circuits, one of which, C, contains the milliammeter A which is to be calibrated and the resistance standard T. The current is adjusted to any desired magnitude by means of the rheostats r_1 and r_2. The slide-wire rheostat r_2 is not necessary but often is useful in making the final adjustment. When the potential difference e across the standard resistance T is measured by means of the potentiometer P, the current in the ammeter circuit and therefore in the ammeter is

$$i = \frac{e}{T} \text{ amp.}$$

This measured value of the current and the milliammeter reading are compared, and the correction for the reading at the given point is determined.

The potential difference across the standard T is measured in the manner described in Exp. 25. The approximate magnitude

of the potential difference across the standard T is first calculated from the ammeter reading and the magnitude of the resistance T; then the appropriate transfer of resistances is made in the boxes R and S of the adjusted potentiometer, and, finally, the experimental test for the balance.

Obtain the corrections for points near every marked division on the scale. Some of the corrections, however, may be determined from observations in the following experiment (28).

Tabulate the observations and corrections, and draw the conventional correction curve.

The temperature coefficient of direct-current ammeters usually is $+ 0.00028/1°F$.

Alternating-current ammeters of the dynamometer, thermocouple, and hot-wire types are calibrated with direct currents. Other types which do not operate with direct currents are calibrated by comparison with any one of the three types that do.

Commercial potentiometers are of various types; all of them, however, usually contain in one box everything that is required to measure potential difference by the standard potentiometer method with the exception of the battery, standard cell, and galvanometer. High-resistance potentiometers usually have in the main circuit a resistance of 10,000 ohms/volt, and the low-resistance potentiometers 50 ohms. The low-resistance potentiometers are especially sensitive when small e.m.fs., such as those of thermocouples, are being measured. In these, part of the spanned resistance is a variable length of resistance wire. One small division on the scale of the drum which carries the contact may represent 0.0000001 volts.

EXPERIMENT 28

CALIBRATION OF AMMETER. MODIFIED POTENTIOMETER METHOD

Apparatus.—The milliammeter used in Exp. 27; apparatus as shown in Fig. 43.

Method and Manipulation.—The resistance of the low-resistance standard T must be large enough so that the potential difference across it is greater than the e.m.f. of the standard cell E_S. The resistance placed in the box R is

$$R = 1000E_S,$$

and the balance is obtained by adjusting the resistance in S. The potential difference across $R + S$ and therefore also across the standard T is

$$e = \frac{R + S}{1000} \text{ volts.}$$

The sum of the currents in T and $R + S$, and therefore the current in the ammeter, is

$$I = i_T + i_{R+S} = \frac{e}{T} + \frac{e}{R + S} = \frac{e}{T} + 0.001 \text{ amp.} \qquad (1)$$

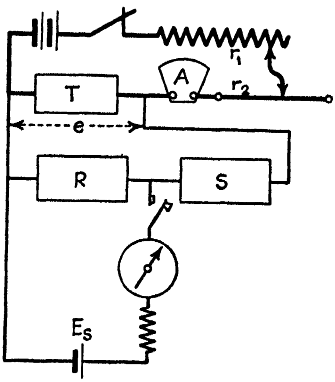


Fig. 43.

The proper trial magnitudes of T, R, and S are determined by calculation and placed in the boxes before the test for balance is made, as is done in the case of all potentiometer measurements.

The chief objection to this useful method is the necessity for keeping the potential difference across the standard T always greater than the e.m.f. of the standard cell. If the whole scale is to be calibrated, several values of T are required; but, if the current to be used does not exceed 0.150 amp, an ordinary resistance box suffices.

Test a sufficient number of points to check the calibration of Exp. 27 and to complete the data for the correction curve.

EXPERIMENT 29

CALIBRATION OF AMMETER. VOLTMETER METHOD

Apparatus.—The ammeter and voltmeter calibrated in Exp. 25 to 28; standard resistance T used in Exp. 27; battery; rheostat; switch.

Method.—The object of this experiment is to test the accuracy of the calibrations made in the foregoing experiments. However, it is often convenient to calibrate an ammeter by means of a calibrated voltmeter.

The resistance of the voltmeter, if not given, must be measured. Then assume that the ammeter is to be the instrument calibrated, and connect it in circuit with the standard resistance T and the rheostat. Span the standard resistance with the voltmeter, and simultaneously read both the ammeter and the voltmeter at several values of the current.

The current in the ammeter at each observed point is

$$I = I_1 + i = \frac{e}{T} + \frac{e}{R_V} \text{ amp,} \tag{1}$$

where e is the corrected voltage reading of the calibrated voltmeter, and T and R_V are, respectively, the resistance of the standard and the voltmeter.

Tabulate the observed readings, the current values determined by Eq. (1), the correction values obtained, and, for comparison, the corrections given for those points by the correction curve of Exp. 27 to 28.

It should be observed that in this and in the potentiometer methods the current is measured in terms of resistance and potential difference and that, whenever it is practical, the ammeter used should have a range such that the readings to be taken are in the upper one-third of the scale.

EXPERIMENT 30

THE CONSTANT OF A MOVING-COIL BALLISTIC GALVANOMETER. ABSOLUTE METHOD

The ballistic galvanometer is used for measuring the quantity of electricity in an instantaneous discharge, such as that from a condenser or a sudden change in the magnetic flux within some part of the galvanometer circuit. It is employed therefore for such measurements as capacitance, mutual and self-inductance, density of magnetic flux, and the magnetic properties of iron.

Any galvanometer may be used as a ballistic galvanometer, but a specially constructed galvanometer is preferable. This has a very broad coil to increase inertia and therefore the period of the ballistic throw. The period is further increased by dispensing with artificial damping. The lengthening of the period is desirable for two reasons: (1) The longer period fulfills more nearly the theoretical requirement that the whole discharge shall take place

before the coil turns an appreciable distance, and (2) the throws are more easily read. The lengthening of the period, however, often has the disadvantage of requiring more time than is necessary for observations. Some investigations in the study of the magnetic properties of iron, however, require very long throw periods; and a galvanometer with a throw period of 58 minutes has been used. A light ring of large diameter, attached to the lower end of the coil at its center, gives the coil a large moment of inertia, which, in conjunction with a long upper suspension, produces the abnormally long period.

The ballistic galvanometer now described and intended to be used in the following experiments is the current-sensitive galvanometer employed in Exp. 21 to 28. Its sensitivity and throw period, however, are increased by the removal of the damping rectangle and by substitution of a 1.5-mil suspension for the one of 3 mils. The removal of the rectangle increases the ballistic-throw sensitivity by approximately the factor 3. The substitution of the finer suspension increases the current sensitivity by approximately the factor 5, the ballistic-throw sensitivity by approximately the factor 2.4, and the period of throw by approximately the factor 2. The zero shift on reversal, however, is usually increased by the factor 7.

A slight damping due to air resistance remains and diminishes each succeeding single vibration by about 6 per cent.

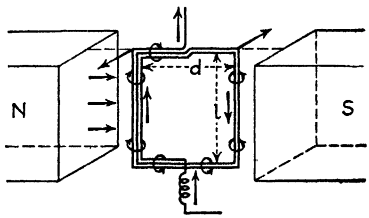


FIG. 44.

I. THEORY

1. Moment of the Force Tending to Rotate the Coil.—When a current I' is energizing the coil, as represented in Fig. 44, each of the two vertical sections of the coil feels a force which tends to turn the coil counterclockwise, as represented.

If B is the density of the magnetic flux, and l and N the length and the number of wires on a vertical side of the coil, the force with which that side is acted on is

$$f = BlNI' \text{ dynes.}$$

When the plane of the coil is parallel to the magnetic lines of force in the open space between the cylinder and the poles of the magnet, the torque acting on the coil is

$$L = BlNdI',$$

where N now represents the number of turns of wire in the coil. In a uniform radial field this torque has the same magnitude in all

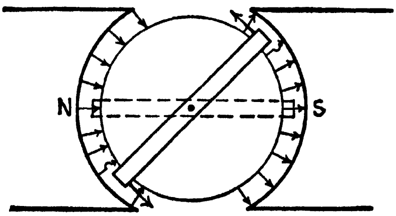


FIG. 45.

positions of the coil if the axis of the coil is in line with the center of the iron cylinder (see Chap. VI, Intr. 1). This is seen by inspection of Fig. 45.

2. The Current Constant K′ of the Galvanometer.—The moment of the force tending to produce rotation must equal the restoring moment $T\phi$ of the torsional couple, when the torques acting on the deflected coil are in equilibrium. Then

$$BlNdI' = T\phi,$$

from which

$$I = 10I' = \frac{10T}{BlNd}\phi = K'\phi,$$

and

$$K' = \frac{10T}{BlNd} = \frac{I}{\phi}. \tag{1}$$

Then

$$\left[BlNd = \frac{10T}{K'}\right]. \tag{2}$$

3. The Quantity Constant K.—Let $\bar{I}'$ be the mean value of the transient current. This current, through electromagnetic reaction, impresses a torque on the coil whose magnitude is

$$L = fr = BlNd\bar{I}' = I_0\bar{\alpha},$$

where I_0 is the moment of inertia of the coil and α the angular acceleration produced by the torque.
Then

$$frt = BlNd\bar{I}'t = I_0\bar{\alpha}t,$$

from which

$$BlNdQ' = I_0\omega,$$

and

$$Q = 10Q' = \frac{10I_0\omega}{BlNd}. \tag{3}$$

This equation gives an expression for the quantity of electricity displaced in the circuit by any transient current, but on the tacit assumption that when the energy of rotation is being acquired by the coil, owing to the torque, it is retained by it completely during the whole period of the discharge.

Each equal element of the discharge contributes more energy to the coil than the one preceding it, because this energy depends on the velocity of the coil at that instant as well as on the torque. If the coil turns an appreciable distance during the period of the discharge, it is transferring an appreciable part of its energy to the suspensions being twisted and to the resisting air. This loss of energy diminishes the velocity of the coil and each increment of the discharge gives less energy to the coil than that assumed in the development of the equation. In order to apply Eq. (3) to the ballistic galvanometer, the period of the throw of the galvanometer must be long enough so that this energy loss during discharge is negligible. What this period is depends on the nature of the discharge. When there is no iron in the circuit, a throw period (one-fourth of a double vibration) of 6 seconds is convenient, in that the readings at elongation are taken easily, but is longer than is necessary. A throw period of 4 seconds is sufficient for most purposes. For large throws, however, this short period may require a mental procedure whereby one fixes in mind, in succession, the final centimeter and millimeter divisions and then, at the instant of the elongation, he concentrates entirely on the estimation to 0.1 of the smallest scale division. The skill to do this is acquired easily. Some have, and perhaps all can acquire with practice, the power to hold a mental picture of the scale

reading long enough after the elongation to enable them to take the throw reading with precision.

Equation (3) gives the quantity of the discharge in terms of the angular velocity ω given to the coil and in other undesirable terms which it is difficult or impractical to measure. It is therefore necessary to substitute for these quantities those which can be measured with comparative ease.

Substituting for $BlNd$ in Eq. (3) its value given in Eq. (2),

$$Q = \frac{10I_0\omega}{10T/K'} = \frac{I_0K'\omega}{T}. \tag{4}$$

The kinetic energy of rotation given to the coil by the instantaneous discharge is $\frac{1}{2}I_0\omega^2$, which, neglecting losses due to damping, becomes the potential energy of the twisted suspensions when the coil reaches the end of the throw. The average torsional moment that the moving coil acts against in its motion to the end of the angle of throw θ is $\frac{0 + T\theta}{2} = \frac{1}{2}T\theta$. The work done by the moving coil in twisting the suspensions = the average torsional moment $\times$ the angular distance $= \frac{1}{2}T\theta \cdot \theta = \frac{1}{2}T\theta^2$. When all the kinetic energy has been converted into potential energy at the elongation,

$$\tfrac{1}{2}I_0\omega^2 = \tfrac{1}{2}T\theta^2,$$

from which

$$\omega = \sqrt{\frac{T}{I_0}} \cdot \theta.$$

Substituting this expression for ω in Eq. (4)

$$Q = \frac{I_0K'}{T}\sqrt{\frac{T}{I_0}} \cdot \theta = \sqrt{\frac{I_0}{T}} \cdot K'\theta. \tag{5}$$

The complete period of vibration of the galvanometer coil $t = 2\pi\sqrt{\frac{I_0}{T}}$, from which $\sqrt{\frac{I_0}{T}} = \frac{t}{2\pi}$. Substituting this expression in Eq. (5),

$$Q = \frac{K't}{2\pi}\theta, \tag{6}$$

in which all the quantities on the right-hand side can easily be measured. It has, however, been assumed that there was no energy lost by the damping of the coil. A correction for this loss must be made as shown in section 4. Also, the period of double vibration t is changed by the damping, but this change is negligibly small in a galvanometer not provided with a damping device.

It should also be observed that since $Q \propto \omega$, and the rotational energy of the coil varies as ω^2, the rotational energy given to the coil varies as the square of the magnitude of the discharge Q.

4. Damping Factor.—Equation (6) is developed on the assumption that all the kinetic energy imparted to the coil by the discharge is changed into potential energy when the coil reaches its elongation. This, however, is not the case. Some of the energy is expended in overcoming the resistance of the air, in the production of induced currents, and in the internal resistance of the suspending fiber. For this reason a correction must be applied to Eq. (6).

The throw of the coil is smaller than it would be if there were no damping. Any observed throw must be multiplied, therefore, by some factor that will increase its value to what it would have been if the coil were undamped.

While the coil is oscillating, the successive angles of single oscillations bear to each other a practically constant ratio. Thus

$$\frac{\Theta_0}{\Theta_1} = \frac{\Theta_1}{\Theta_2} = \frac{\Theta_2}{\Theta_3} = \frac{\Theta_{(n-1)}}{\Theta_n} = \rho \text{ nearly,}$$

from which

$$\Theta_0 = \rho\Theta_1 = \rho^2\Theta_2 = \rho^3\Theta_3 = \rho^n\Theta_n. \tag{7}$$

For example, to find what the angle Θ_2 would have been had there been no damping in the last two vibrations, Θ_2 is multiplied by ρ^2, *i.e.*, by ρ *with an exponent numerically equal to the number of intervening single oscillations.*

When an actual throw is taken, the coil starts from the center of its natural arc of oscillation and moves, therefore, through but *one-half* of a single oscillation. To find what the angle of throw would have been without damping, as has just been shown, the observed angle of throw θ_1 must be multiplied by ρ with an

exponent numerically equal to the number of intervening vibrations, *i.e.*, ½. Then

$$\theta = \rho^{1/2}\theta_1. \tag{8}$$

Substituting this in Eq. (6),

$$Q = \frac{K't\rho^{1/2}}{2\pi}\theta_1 = K\theta_1, \tag{9}$$

from which the constant

$$K = \frac{K't\rho^{1/2}}{2\pi}. \tag{10}$$

This constant is for open-circuit work only. The damping factor $\rho^{1/2}$ is a correction for air damping and should not be greater than 1.03 because the laws of air damping are not definitely known. Highly damped ballistic galvanometers have electromagnetic damping whose laws are known and do not change the relation $\theta \propto Q$. The constants of such highly damped galvanometers must be determined by the comparison methods of Exp. 32. These comparison methods are used normally in any case, because of their comparative simplicity. The absolute method is given here for obtaining a better understanding of the galvanometer and for showing how the magnitudes of standards used for comparison purposes can be determined by an absolute method.

Magnetic impurities within the coil influence[1] somewhat the magnitude of the throw. Equation (8) makes no correction for this effect which usually is not large. The moving-magnet type of the galvanometer is free from this source of error, and therefore its constant can be determined with greater precision by the absolute method.

The temperature coefficient of the moving-coil galvanometer for ballistic throws is about -0.00017 per 1°C.

II. DETERMINATION OF THE CONSTANT

The constant $K = \dfrac{K't\rho^{1/2}}{2\pi}$ is obtained by determining separately the values of K', t, and $\rho^{1/2}$.

Apparatus.—The apparatus is connected as shown in Fig. 46, where M is an open coil of the proper number of turns and a

[1] *Phys. Rev.*, Vol. 23, p. 297, 1911.

magnet of sufficient strength which when moved within the coil may induce an e.m.f. large enough to check quickly the oscillations of the galvanometer coil. The resistance T may be composed of several parts to make a total resistance of about 20,000 ohms.

1. Determination of K′.—Make $R = 5$ or 10 ohms, and S such (usually several hundred ohms) that with a nearly full-scale reading on the voltmeter the galvanometer deflects about 5 or 6 cm.

The voltmeter gives the potential difference E across $R + S$. Then when e is the potential difference across R and the galvanometer branch,

$$\frac{e}{E} \cong \frac{R}{R + S},$$

from which

$$e = \frac{R}{R + S} E \text{ volts.}$$

Then the current in the galvanometer is

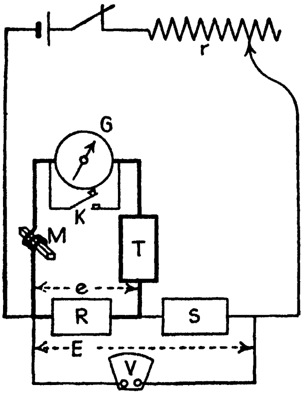


FIG. 46.

$$i = \frac{e}{T + G + M} \text{ amp,}$$

where $T + G + M =$ the total resistance in the galvanometer branch.

The current constant then is

$$K' = \frac{i}{\phi} \text{ amp/radian.} \tag{11}$$

where $\phi = \frac{d}{2D}$ radians [see Chap. VI, Exp. 16(3)].

Because of the high resistance T, the galvanometer is underdamped; and because the null reading on reversal of deflection changes, the coil must be brought to rest without crossing the null point; it is necessary therefore to bring the coil to rest after a deflection by short-circuiting it with the key K. Also, when the galvanometer coil is deflected it swings beyond the point of equilibrium and then oscillates about it. The first swing may be checked and the oscillations damped by moving the damping

magnet properly within the coil M. The deflection d is measured from the null reading taken immediately after the deflection.

The constant K' [Eq. (11)] is the current constant at the instant when the transient current is deflecting the coil. If the magnetic field is not uniform or the galvanometer coil is not hanging centrally, this value may differ considerably from that at the deflected position. It is for this reason that the deflections taken are as small as is consistent with the percentage observational error in the measurement of the distance from the scale to the axis of the coil. Also, it is necessary to take a deflection with the same current in the reverse direction. If the field varies, the average magnitude of the oppositely directed deflections is more nearly that which would be obtained if the field and the adjustment were perfect.

On bringing the coil to the null point after a deflection the short-circuiting greatly overdamps the coil, so that unless the circuit is closed at the proper instant it is necessary to repeat the opening and closing of the short-circuiting key.

Finally, the short-circuited coil comes to rest, but at a point other than the true null point. A thermoelectric current almost invariably deflects the coil, so that on opening the circuit the coil again oscillates. If, now, the coil is again short-circuited for an instant as it is crossing the null point in its motion away from its closed-circuit position of rest, the impulse given the coil by the themoelectric current checks the momentum. In this manner the amplitude of oscillation is reduced quickly by an amount depending on the skill of the operator. Then in place of waiting for the coil to come to rest, the null reading may be determined from the observed readings at three or five successive elongations. In a similar manner the deflected reading may be determined from the readings at three or five successive elongations.

2. Determination of t.—Cause the coil to oscillate by deflecting it with a small current and then *opening* the circuit. Determine t by the method of [Chap. VI Exp. 8(1)]. The passages of the null reading on the scale are observed in the telescope. A wire may be hung over the null-point division, if necessary, to make the passages visible. A sufficiently large current may be obtained by holding the wire leads in moistened fingers or by disconnecting

the galvanometer branch from the main circuit (Fig. 46) when the coil has a deflection of about 12 cm. This measurement requires two observers.

3. Determination of $\sqrt{\rho}$.—Cause the coil to oscillate an open circuit, and record the scale readings at a succession of elongations, to right and left, beginning with one of about 12 cm and ending with 1 cm. Then from Eq. (7), on a circular scale

$$\frac{d_0}{d_n} = \frac{\Theta_0}{\Theta_n} = \rho^n,$$

from which

$$\sqrt{\rho} = \sqrt[2n]{\rho^n}. \tag{12}$$

The terms d represent the observed lengths of any two arcs, *i.e.*, the distance between the extremes of each oscillation and not the distance from the zero point to one extreme. And n represents the number of oscillations through which the coil has been damped since the first oscillation d_0.

In a moving-coil galvanometer $\sqrt{\rho}$ varies somewhat with the amplitude of vibration. It is therefore desirable to obtain its value for nearly the same amplitude as that of the throw in any particular case. This can be obtained from only a few oscillations immediately following a throw.

EXPERIMENT 31

THE CONDENSER CHARGE AND CAPACITANCE. ABSOLUTE METHOD

Apparatus.—Mica condenser; ballistic galvanometer whose constant was determined by the absolute method in Exp. 30; condenser discharge key; short-circuiting tap key; a resistance of 10,000 to 30,000 ohms to be used for protecting the standard cell.

The mica condenser consists of many thin sheets of tin foil insulated from each other by thin sheets of mica. All alternate sheets of the tin foil are joined together to form the two sets of a many-plate condenser. The whole is immersed in a hot insulating compound and allowed to cool under great pressure. Such a condenser has a comparatively small absorbed charge and a temperature coefficient varying from -0.00005 to $+0.00002/1°C.$

The charges on each plate of a condenser consist of four parts: (1) the true-free charge which is present on the plate to maintain its impressed potential and which discharges if the plate alone is grounded. This charge is only a small fraction of the whole condenser charge and is represented by the uninclosed + and − signs in Fig. 47; (2) the charge bound on the plate by the charge on the other plate represented by the signs ++ and − −; (3) the charge bound on the plates by the atomic doublets *DD* of the polarized dielectric. The bound charges (2), (3) are by far the greater part of the condenser charge. The three parts (1)(2)(3) of the total charge, together, are called the *free charge* of the condenser. The true-free charge (1) is present on only one of the plates if the other plate had been grounded after or at the time when the condenser was being charged. The total charge on one of the plates, in such a case, is slightly larger than that on the other. When the condenser is being discharged, electrons are displaced in the circuit until the potentials of the plates are equalized. The charges of the atomic doublets of the dielectric reunite freely as the electric field F'' between the plates diminishes, so that the discharge follows a definite law for a free charge as given in Exp. 40; also, the whole measurable part of the charge is displaced through a small resistance in less than 0.01 seconds.

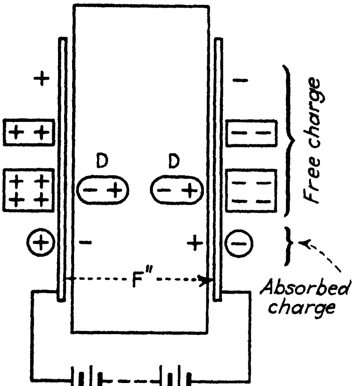


FIG. 47.

This displaced charge, equal in magnitude to the average charge on the plates, can be measured by a ballistic galvanometer and, with the potential difference across the plates, determines the capacitance of the condenser.

Material dielectrics are not perfect dielectrics, and consequently some free electrons migrate through them toward the positive plate. This migration causes a separation of the charges within the dielectric as represented by the lower + and − signs, which, owing to surface conditions, is not entirely compensated by an

equal flow to and from the plates. The whole dielectric therefore slowly becomes an electric doublet, which, like the atomic doublets, increases the bound charges on the plates as represented by the encircled + and − signs and labeled "absorbed charges" in Fig. 47. When the condenser is discharged, the atomic doublets and the electric field F'' disappear, but the dielectric doublet remains and continues to bind the "absorbed charges" on the plates. These absorbed charges, therefore, do not contribute a significant part to the instantaneous discharge of a condenser. However, the absorbed charge is gradually liberated, often for many hours, as the displaced electrons of the dielectric slowly migrate toward the + charges, so that a measurable part

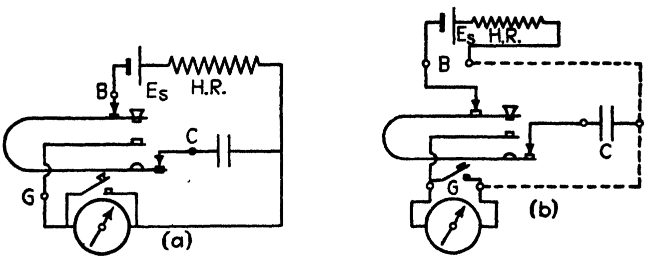


FIG. 48.

of this bound charge is liberated during the comparatively long period of the galvanometer. It is therefore the practice to disconnect the condenser as soon as the whole measurable part of the free charge has been discharged and before a measurable part of the absorbed charge has been liberated. This is conveniently accomplished, nearly enough, by means of a special discharge key shown in the diagram of Fig. 48. One tap on the key depresses the three springs in succession and in succession opens the charging-battery circuit, discharges the condenser for about 0.01 seconds, and then disconnects the condenser.

The contact points in such a key often give trouble but may be cleaned by means of a slightly roughened strip of steel drawn through them under pressure. Such strips are used by telephone companies for a like purpose.

Method and Manipulation.—Connect the apparatus as shown in Fig. 48. The high resistance *HR* does not change the charging time enough to be noticeable and aids in protecting the standard

cell. However, the student who is using the key for the first time should make a preliminary test with a storage cell in place of the standard cell E_S. It should be observed that in the key of Fig. (*a*) one side of each of the three main instruments is connected to an appropriately labeled binding post on the key, and the other binding post of each of the three instruments is connected to a common point; *i.e.*, these three binding posts are connected together. In the form of the key shown in Fig. (*b*) the three main instruments are connected each to its two appropriately labeled binding posts on the key. No further connections are required,

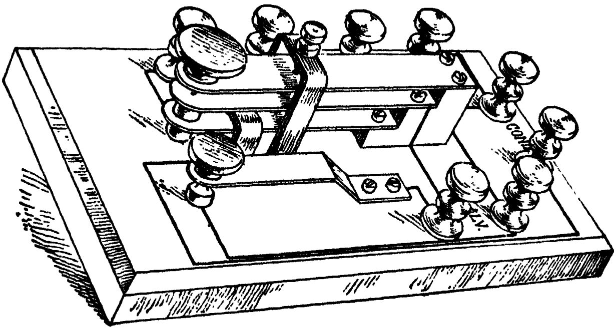


FIG. 49.

for the second three binding posts are now permanently connected within the key as represented by the broken lines. This form of the key is also shown in Fig. 49.

Practice taking throws until consistent readings are obtained agreeing to within 0.01 cm for throws up to 5 cm and to within 0.02 cm for larger throws. Difficulty will be experienced in bringing the coil to rest at the null point, but there should be none in reducing the oscillations to within 1 mm to either side of the null point. Throws are then taken at the instant when the coil reaches the elongation on the side of the throw. The coil then is at rest and thereby fulfills the necessary requirement for the taking of ballistic throws. The throw, however, must be measured from the null point and not from the elongation. The throw reading is the same regardless of whether the throw is taken from the null point or from the elongation, provided the elongation is not too far from the null point.

Let D (Fig. 50) represent the ballistic throw when taken from the null point. When taken from the elongation at a distance d from the null point, the throw extends beyond that obtained from the null point by an amount e. This error e is shown to be negligible.

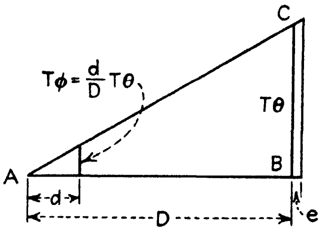


FIG. 50.

Let $T\theta$ be the torque of the suspensions at the throw point B; and $T\phi = \dfrac{d}{D}T\theta$, the torque at the elongation d. In either case the discharge gives the coil the same rotational energy represented by the potential energy of the twisted suspensions in the triangle ABC. When the throw is taken from the elongation, therefore, the coil still has kinetic energy when it reaches the correct throw reading at the point B. This energy is represented by the small triangle whose base is d and in arbitrary units has a magnitude

$$w = \frac{1}{2}d \cdot T\phi = \frac{d}{2} \cdot \frac{d}{D}T\theta = \frac{d^2}{2D}T\theta.$$

This energy carries the coil beyond the point B a distance e such that the energy expended in moving that distance is

$$w \cong e \cdot T\theta = \frac{d^2}{2D}T\theta,$$

from which the error

$$e = \frac{d^2}{2D}\text{cm}, \tag{1}$$

when d and D are both measured in centimeters. This is shown to be negligible, for when $d = 0.1$ cm and $D = 5$ cm,

$$e = \frac{d^2}{2D} = \frac{0.01}{10} = 0.001 \text{ cm},$$

which cannot be detected on the scale.

Remembering that the first throw is always discarded; that the key must be held in the depressed position until the throw is taken; that the coil is not allowed to cross the null point; that the throws

may be taken from an elongation, but must always begin when the coil is at rest and be measured from the true null point; that in case the null reading changes, the one taken before the throw is used; and that the null point may be determined from three or five elongations, take five consistent throws. Then

$$Q = K\theta_1 = K\frac{d}{2D} \text{ coulombs,} \tag{2}$$

where d and D now are respectively the observed throw and the distance between the scale and the galvanometer coil,
and

$$C = \frac{Q}{E_S} \text{ farads.} \qquad C_{\mu f} = \frac{10^6 Q}{E_S} \text{ microfarads.} \tag{3}$$

Compare the determined magnitude of the capacitance with its certified value. Also, calculate the magnitude of the capacitance in micromicrofarads and in statfarads.

EXPERIMENT 32

THE CONSTANT AND FIGURE OF MERIT OF A BALLISTIC GALVANOMETER ON OPEN CIRCUIT. STANDARD CONDENSER METHOD

Apparatus.—The apparatus of Exp. 31, except that now the certified value of the capacitance of the mica condenser is assumed. The observations of Exp. 31 may be used for the determination of the constant and the figure of merit.

Method.—When the capacitance and the e.m.f. are known,

$$Q = CE = K\theta_1 = K\frac{d}{2D} = Fd,$$

from which

$$K = \frac{CE}{\theta_1}, \qquad \text{and} \qquad F = \frac{CE}{d} \text{ coulombs/cm.}$$

F = figure of merit, *i.e.*, the quantity of electricity that produces a throw represented by 1 cm on the scale,
and d = throw in cm.

This is an indirect method and is the one most commonly employed when the galvanometer is to be used on open circuit.

The *figure of merit* replaces the *constant*, especially when a circular scale is used. The millimeter is employed by some in place of the centimeter as the unit of throw, but the centimeter will be used in this text. A new determination of the value of the figure of merit should be made at the time of each experiment.

The term *figure of merit* is also employed to represent the capacitance per centimeter throw for a given charging e.m.f.

$$C = \frac{Q}{E} = \frac{Fd}{E} \text{ farads.} \qquad C_{\mu f} = \frac{10^6 Fd}{E} \text{ microfarads.}$$

But $10^6F/E$ is a constant when the same e.m.f. is employed. Let f represent this constant. Then

$$C_{\mu f} = fd.$$

It is seen that f is a *figure of merit* which when multiplied by the throw gives the capacitance in microfarads.

The *figure of merit* is used only when many quantities or capacitances are measured at the same time.

When only one such quantity is measured, since Q or $C \propto d$, the unknown and the known are to each other directly as the throws. The capacitances vary as the throws only when the same charging battery is used; otherwise the e.m.fs. must be known and used as follows:

$$C_xE_x = Fd_x,$$
$$CE = Fd,$$

from which

$$\frac{C_xE_x}{CE} = \frac{d_x}{d}.$$

EXPERIMENT 33

CRITICAL-DAMPING RESISTANCE

The critical-damping resistance is that which when introduced into the circuit with the galvanometer makes the galvanometer just aperiodic on closed circuit; *i.e.*, the deflection or throw reaches the deflected position in the minimum length of time without crossing it and returns to the null reading in the same manner. Any additional resistance would reduce the damping so that the coil would move beyond these points.

Method and Manipulation.—The apparatus of Fig. 48, except that now a storage battery is used and one side of the galvanometer is also connected through a resistance box to a binding post usually marked "App," which closes the galvanometer through any desired apparatus, in this case through a resistance box, immediately after the condenser discharge. This connection is shown in a modified form in Fig. 65. The throw should be one of about 10 cm and may require the use of a paper condenser for its larger capacitance, because the galvanometer is much less sensitive when on closed circuit. If a sufficiently large throw cannot be obtained with the apparatus available, place a tap key into the connection AB, Fig. 65. This allows the coil to move to the elongation on open circuit, and then by closing this key one makes the coil return to the null point on closed circuit.

In either case the resistance is found where the coil just crosses the null reading. A slightly smaller resistance is that which makes the motion just aperiodic. The critical resistance can be determined readily to within 5 or 10 ohms and for ordinary laboratory galvanometers varies from 200 to 600 ohms. A more accurate determination serves no purpose.

EXPERIMENT 34

CORRECTION CURVE FOR THROW PROPORTIONALITY ON OPEN CIRCUIT

Owing mainly to the imperfection of the circular scale and to its adjustment with respect to the axis of the coil, the ballistic-galvanometer throws are not exactly proportional to the quantities of electricity even on open circuit. The accuracy of the proportionality is sufficient for most practical work and rarely introduces an error of more than 1 per cent even when the smallest throws are compared with those of the greatest magnitude. For more accurate comparisons one of the quantities of electricity is made such that the two give nearly equal throws on the galvanometer, or the observed throws are corrected for disproportionality by means of a correction curve.

Method and Manipulation.—By means of the apparatus shown in Fig. 51, a mica condenser C is charged with any desired potential difference impressed across the resistance R. When the total resistance in the battery circuit is kept constant, and the resistance

in R is varied by an interchange with S, the potential difference across R is proportional to the resistance in R. The ballistic throws given by the condenser discharges, therefore, should be proportional to the resistances in R. It is desirable to compare all throws with that nearest the 10 cm division, for that is considered to be the optimum throw and can be read to within 0.1 per cent.

Use the proper number of storage cells and a mica condenser of sufficient capacitance so that when $R = 1000$ and $S = 1500$ ohms the condenser discharge gives a throw of about 10 cm. Then adjust the rheostat r until the throw is as nearly 10.00 cm as it is convenient to make it. Hereafter the total resistance in the circuit is kept constant unless the 1000-ohm resistance in R gives a throw of a changed magnitude. Then by interchange with S make $R = 2500$. The throw reading then should be 25 cm, but the object of the throw is to prevent as far as possible a change in zero shift during the experiment. A large zero shift usually means that the coil is not properly centered with respect to the radial magnetic field and may be reduced by a proper leveling of the galvanometer. The null point now is readjusted to read zero; and with $R = 1000$, r is readjusted, if necessary, to give the throw of 10.00 cm. The apparatus, then, is ready for the final observations.

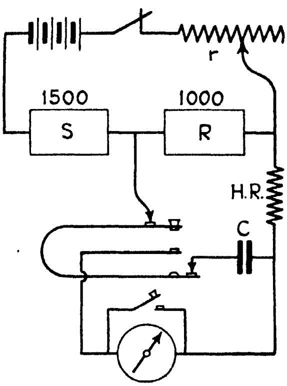


FIG. 51.

Sets of observations are taken with the resistance in R changed for each succeeding set by 100 ohms, first from 1000 to the lowest and then from 1000 to the highest. It is desirable to check the standard throw at intervals between the sets. The procedure in the taking of observations given in Exp. 31 is followed always.

The correction to be applied to any scale reading is obtained as follows: Assume the standard throw, for example, to be 10.12 cm in place of the exact 10.00. Then for each 100 ohms in R the throw should be 1.012 cm; and with 800 ohms in R the throw should be $1.012 \times 8 = 8.10$ cm. If the observed throw is 8.13 cm, the throw at 8.13 is too large by 0.03 cm, and, therefore, the correction to the scale at that point is -0.03 cm.

Tabulate (1) the calculated throws, (2) the observed throws (corrected for null reading), and (3) the calculated corrections with their proper signs, and draw the correction curve (see Chap. VI, Exp. 1). This correction curve should be used in all future experiments when the coil moves on open circuit, except when the throws being compared are nearly equal. If a sectional standard condenser is available, such a condition is easily attained and is to be preferred.

The correction curve for closed-circuit work may differ from that on open circuit because of a disproportionality in the large amount of damping. Such a correction curve is obtained in Exp. 35; but the correction curve for open-circuit work makes a partial correction for the disproportionality of throws on closed-circuit work. It improves the proportionality when applied to such work and therefore should be used when the proper correction curve is not available.

EXPERIMENT 35

CORRECTION CURVE FOR THROW PROPORTIONALITY IN A CRITICALLY DAMPED, CLOSED CIRCUIT

Apparatus.—Cenco variable magnetic standard or a mutual-inductance standard connected in circuit as shown in Figs. 52(*a*), (*b*). The resistance $R + S$ in the circuit is that which gives the critical damping and whose magnitude was determined in Exp. 33.

Method and Manipulation.—1. The Cenco variable standard S [Fig. 52, (*a*)] consists of a coil held by strong spiral controls in a radial magnetic field. The coil can be brought into any desired marked positions where it is held automatically. When released, it springs back to the null position, causing a known change in its flux turns. This change

$$N\phi = 10^8 RQ = Kd. \tag{1}$$

Therefore the galvanometer throws should vary as the flux-turn change in the standard.

2. The mutual-inductance standard consists of two coils held in a common frame which are represented by the two adjacent coils S [Fig. 52(*b*)]. When the battery circuit is opened the e.m.f

induced in the coil, called the secondary coil, which forms a part of the galvanometer circuit is

$$e = -M\frac{di}{dt} \equiv M\frac{I_P}{t} = R\bar{\imath},$$

from which

$$I_P = \frac{R\bar{\imath}t}{M} = \frac{R}{M}Q = K_1Q = Kd. \qquad (2)$$

Therefore the throw d should vary as the current change in the primary circuit.

In either case 1 or case 2 all throws are compared with the one whose magnitude is as near 10.00 cm as it is practical to make it.

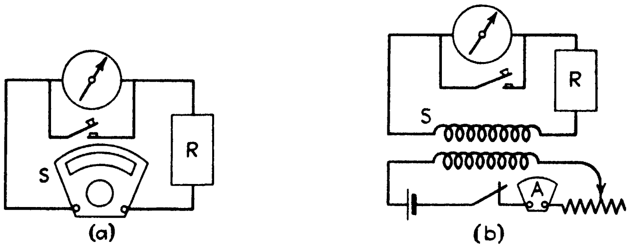


FIG. 52.

When the standards are being reset, the galvanometer must be short-circuited to prevent a reverse throw. Test convenient points along the whole galvanometer scale, and tabulate the observations and the calculated corrections as in Exp. 34. Draw the correction curve for the critically damped closed circuit on the same sheet with that for the open circuit.

On the closed as well as on the open circuit it is preferable, whenever convenient, to make the throws to be compared nearly equal and, therefore, the use of the correction curve unnecessary.

EXPERIMENT 36

THE AYRTON SHUNT

In Fig. 53 the galvanometer is in circuit with two resistances R and S. The discharging condenser may be connected either across R or across $R + S$. It will be shown that if $R = S$, for example, regardless of what the resistance of the galvanometer may be, the part of the total discharge from a condenser through

the galvanometer when the condenser is discharging across $R + S$ is twice that when the condenser is discharging across R. The throws from the same condenser discharge are to each other in the two cases as $R + S$ is to R. This is true whatever may be the magnitudes of the resistances. Let $\frac{R + S}{R} = n$. Also, let d_1 be the observed throw produced by the quantity q_1 that passed through the galvanometer while the quantity Q_1 in the condenser when connected across R is discharged. Let d_2, q_2, and Q_2 represent corresponding quantities when the condenser is connected across $R + S$. Then

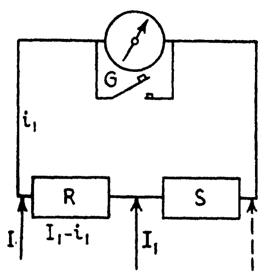


FIG. 53.

$$\frac{nd_1}{d_2} = \frac{nq_1}{q_2} = \frac{Q_1}{Q_2}. \tag{1}$$

The shunt is used principally for comparing large condenser discharges with small ones.

Let i_1 be the average magnitude of the current that energizes the galvanometer coil when the discharging condenser is connected across R; and I_1, the average total current. The self-inductance in the galvanometer, whose resistance is G, does not influence these averages. Then

$$\frac{i_1}{I_1 - i_1} = \frac{R}{S + G},$$

from which

$$i_1 = \frac{R}{R + S + G} I_1.$$

Multiplying both sides by the time of discharge gives

$$q_1 = \frac{R}{R + S + G} Q_1. \tag{2}$$

When the condenser is connected across $R + S$, similarly

$$\frac{i_2}{I_2 - i_2} = \frac{R + S}{G},$$

from which

$$i_2 = \frac{R + S}{R + S + G} I_2,$$

and

$$q_2 = \frac{R + S}{R + S + G} Q_2. \tag{3}$$

From Eqs. (2)(3) and the proportionality of the throws

$$\frac{d_1}{d_2} = \frac{q_1}{q_2} = \frac{R}{R + S} \cdot \frac{Q_1}{Q_2} = \frac{1}{n} \cdot \frac{Q_1}{Q_2},$$

from which

$$\frac{Q_1}{Q_2} = \frac{nd_1}{d_2}. \tag{4}$$

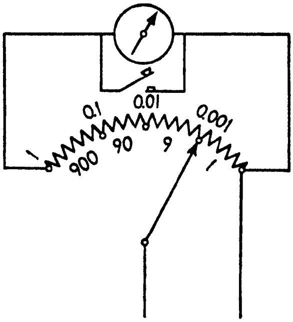


FIG. 54.

An Ayrton shunt can be made from any two resistance boxes, and n can have any value.

A diagram of a commercial form of the Ayrton shunt, shown in Fig. 54, has the connections from the condenser span 0.001 of the whole resistance of the shunt. The observed throw multiplied by 1000 gives the throw the same condenser discharge would produce if it were connected across the whole resistance of the shunt. By moving the arm of the switch to the other three lugs on the shunt, the positions requiring the factors 100, 10, and 1 are obtained in succession.

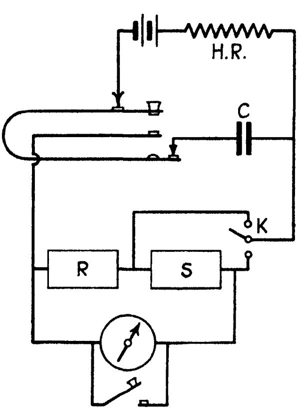


FIG. 55.

Make an Ayrton shunt of two resistance boxes in which the sum $R + S$ of the two resistances is that which gives critical damping and at the same time $\frac{R + S}{R} = n = 2$. Then compare the corrected throws obtained from equal condenser discharges across R and $R + S$. The connections may be made as shown in Fig. 55.

The Ayrton shunt diminishes greatly the sensitivity of the galvanometer, (1) because part of the discharge is diverted through the shunt, and (2) because the critical damping diminishes the sensitivity of the galvanometer by a factor of about 2.7.

A larger maximum sensitivity with the shunt is obtained, therefore, when a larger than critical resistance is employed. This, however, must not be so high as to increase the period of the condenser discharge enough to affect the throw appreciably.

An Ayrton shunt made of two resistance boxes is more flexible than the commercial type of Fig. 54.

EXPERIMENT 37

CAPACITANCE OF A PAPER CONDENSER. COMPARISON METHOD

Apparatus.—The apparatus is that of Exp. 31, shown in Fig. 48, except that the standard cell is replaced by an ordinary cell; paper condenser; mica condenser; Ayrton shunt; s.p.d.t. switch.

Method and Manipulation.—1. Charge the standard mica condenser, and then discharge it through a ballistic galvanometer. In the same manner charge and discharge the paper condenser. Then from the standard

$$Q = CE,$$

and from the paper condenser

$$Q_x = C_xE,$$

from which, since the charging e.m.f. is the same for both condensers,

$$\frac{C}{C_x} = \frac{Q}{Q_x} = \frac{d}{d_x},$$

and

$$C_x = \frac{d_x}{d}C \tag{1}$$

where C_x and C in Eq. (1) may also represent the capacitances in microfarads. The charging e.m.f. must be of such magnitude that both condensers give throws of appropriate magnitude. A potential difference across a resistance may be used in place of the whole e.m.f. of a cell or battery and has the advantage of being adjustable to any desired magnitude. The standard cell is not used (*a*) because it is not necessary to know the magnitude of the charging e.m.f., and (*b*) because the paper condenser often has a

low resistance and a large absorbed charge, subjecting the cell thereby to a large unnecessary drain.

2. Also determine the capacitance of the same paper condenser using the Ayrton shunt with the factor n such as to make the throws from the two condensers nearly equal. It then is not necessary to use the correction curve.

Compare the results obtained by the two methods.

Paper condensers often hold large absorbed charges, and telephone and transmission cables, always. Figure 56 shows the difference in the discharges of two condensers of equal capacitance, one of which, M, has a negligible amount of the absorbed charge liberated in the first 0.13 seconds, while the other, P, has a considerable amount. In condenser M there is no appreciable difference in the throw regardless of whether the time of discharge is 0.01 or 0.03 seconds, while with the condenser P there is. The throws taken from condenser P, therefore, are likely to differ in magnitude if the consecutive time periods during which the condenser is allowed to discharge differ. It is desirable then to tap the key with greater speed, in order that the condenser may be disconnected from the galvanometer as soon after the passage of the free charge as possible.

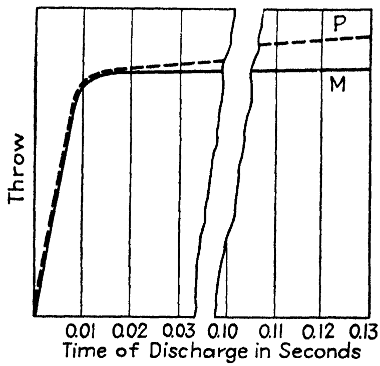


Fig. 56.

EXPERIMENT 38

RESIDUAL CHARGES IN A CONDENSER

Apparatus.—The apparatus of Exp. 31, 37.

Method and Manipulation.—After a condenser is discharged, it gradually acquires an appreciable charge due to the liberation of the "absorbed" charge (see Exp. 31). This charge is usually larger in paper than in mica condensers but is small in all good condensers. In telephone cables the amount of the absorbed charge liberated during the throw period of the galvanometer may be equal to 15 per cent of the free charge.

Connect a paper condenser to a condenser discharge key in the same manner as in Fig. 48.

Charge the condenser for 5 minutes, and then permanently open the battery circuit. Discharge the condenser at a known time, and keep the discharge key down. Bring the galvanometer coil to rest, and at the end of 2 minutes release the key. The charge liberated during the 2 minutes will then discharge through the galvanometer. At the end of another 2 minutes press down the key, and obtain the second residual discharge. Continue in this manner until several discharges have been taken.

Plot the throws for the residual discharges.

If the residual charges are small, their magnitude may be increased by increasing the charging e.m.f. The original free charge will then be too large for the galvanometer. The galvanometer then should be short-circuited during the original discharge.

Why is it necessary to charge the condenser for so long a period?

EXPERIMENT 39

ELECTROMOTIVE FORCE OF A CELL. CONDENSER METHOD

Apparatus.—The apparatus of Exp. 31; a cell whose e.m.f. is to be measured; standard cell.

Method and Manipulation.—Charge a mica condenser with the cell whose e.m.f. is to be determined, and discharge it through a ballistic galvanometer. Also, charge the same condenser with a standard cell, and discharge. Then

$$Q_x = CE_x,$$

and

$$Q = CE,$$

from which

$$\frac{E_x}{E} = \frac{Q_x}{Q} = \frac{d_x}{d},$$

and

$$E_x = \frac{d_x}{d}E. \tag{1}$$

Any potential difference may be determined in this manner. It is, therefore, possible to measure a current by this method in terms of resistance and e.m.f. or a resistance by the fall-of-

potential method. It is especially useful when it is necessary to measure the potential difference across a very high resistance.

EXPERIMENT 40

INSULATION RESISTANCE. METHOD OF LEAKAGE

Apparatus.—The apparatus of Exp. 31; a very high resistance, such as the insulation between the two coils of a standard mutual inductance.

Method.—Let Q_1 be the original charge in the condenser, and let Q_2 be the charge left in it after the condenser has been discharging through a high resistance or insulation for a time interval t. Also, let C be the capacitance of the condenser, and i the instantaneous value of the discharge current in the high resistance. Then

$$i = -\frac{dq}{dt}.$$

Also, because at any instant $q = Ce$,

$$i = \frac{e}{R} = \frac{q}{CR}.$$

$$\therefore \frac{q}{CR} = -\frac{dq}{dt},$$

from which

$$\frac{1}{R}\int_0^t dt = -C\int_{Q_1}^{Q_2}\frac{dq}{q}.$$

$$R = \frac{t}{C\log_e\dfrac{Q_1}{Q_2}} = \frac{t}{2.3026C\log\dfrac{Q_1}{Q_2}} = \frac{t}{2.3026C\log\dfrac{d_1}{d_2}} \qquad (1)$$

where C is the capacitance in farads.

When the condenser discharges through itself only, the resistance R is the resistance R_c of the condenser. When the unknown resistance R_x is connected so that the condenser discharges through it also, the resistance R in Eq. (1) is the resistance of the condenser and the resistance R_x in parallel. Then

$$\frac{1}{R} = \frac{1}{R_c} + \frac{1}{R_x},$$

from which

$$R_x = \frac{RR_c}{R_c - R} \text{ ohms.} \qquad (2)$$

Manipulation.—The magnitude of d_1 produced by the discharge of the whole free charge of the condenser is obtained in the standardized manner of Exp. 31. The throw d_2, due to the charge left in the condenser after the condenser has been discharging for a known period of time through a resistance, requires special consideration. The displacement of the electrons in the dielectric which produces and holds the absorbed charge on the plates continues to increase until a condition of equilibrium is established even after the condenser has been disconnected from the charging battery. In this manner some of the free charge becomes a part of the absorbed charge. Therefore, when the condenser is discharged after the interval t, the throw d_2 is too small by an amount proportional to the charge absorbed during that period. This loss, however, is compensated in part or even overcompensated by the liberation of some of the original absorbed charge due to the diminution of the potential difference across the plates during that interval. It is observed from the foregoing that, especially when the absorbed charges are large, the magnitude of d_2 does not represent accurately the residue of the original free charge. To minimize this effect of the absorbed charge, the condenser is charged for 5 minutes to enable the absorbed charge to reach approximately its state of equilibrium before the charging battery is disconnected and the condenser is allowed to discharge through the resistance to be measured.

The best value of the time interval t is that which makes $d_2 = \frac{1}{2}d_1$. This is rarely attempted, because high precision is impossible and is not required in the measurement of such high resistances. For obtaining d_2, the charging circuit is opened by disconnecting the wire from the battery, because the ordinary switch, when open, may cause a partial charging of the condenser through the insulation of the switch during so long a period. Ten minutes for the time of discharge t is usually sufficient but may have to be less.

When the high resistance to be measured is connected across the condenser, the condenser may be charged with the resistance attached.

If the resistance to be measured is that of the insulation between two coils or between two wires in a telephone cable, connecting these across the condenser adds to the capacitance. Then the capacitance C [Eq. (1)] is the capacitance, in parallel, of the condenser and the two parallel wires of the coils or cable.

Measure (1) the resistance R_c of the mica condenser, (2) the capacitance of the mica condenser together with the two parallel wires whose insulation resistance is to be measured, (3) the resistance R of the condenser and the insulation in parallel. Then calculate the resistance R_x of the insulation, using Eq. (2).

EXPERIMENT 41

INSULATION RESISTANCE. DIRECT-DEFLECTION METHOD

Apparatus.—The ballistic galvanometer shunted for critical damping or any sensitive galvanometer; the two coils of Exp. 40 whose insulation resistance is to be remeasured; a known resistance of several megohms; a battery of such an e.m.f. as to produce a measurable deflection on the galvanometer through the insulation and not too large a deflection through the known high resistance.

Method.—1. Obtain the deflection d_x through the resistance R_x in series with the battery and galvanometer. Then substitute the known high resistance R for the unknown R_x, and obtain the deflection d. Since the galvanometer resistance is negligible,

$$\frac{R_x}{R} = \frac{d}{d_x},$$

from which

$$R_x = \frac{d}{d_x} R \text{ ohms.} \tag{1}$$

When the deflection d_x is to be taken, the galvanometer should be short-circuited at the time when the battery circuit is being closed. This short-circuiting prevents a throw due to the charging of the distributed capacitance of the two parallel wires of the coils.

2. With R_x in circuit with a battery of known e.m.f. and the unshunted galvanometer whose current constant K_1 was determined in Exp. 30,

$$R_x = \frac{E}{i} \text{ ohms.} \tag{2}$$

The sensitivity of the galvanometer is

$$f = \frac{K_1}{2D} \text{ amp/cm},$$

and

$$i = fd \text{ amp.}$$

Compare the values of the insulation resistance with that determined in Exp. 40.

EXPERIMENT 42

THE CONSTANT OF A SOLENOID

The intensity of a magnetic field can be measured by the work required to move a unit point pole in it through a distance of 1 cm in the direction of the lines of force. Imagine such a unit point pole placed at the center of a solenoid (Fig. 57). This pole has emanating from it 4π lines of force. If the solenoid is of sufficient length so that all these lines can be assumed to penetrate its loops of wire, the work done in moving the point pole a distance equal to the thickness of the wire is

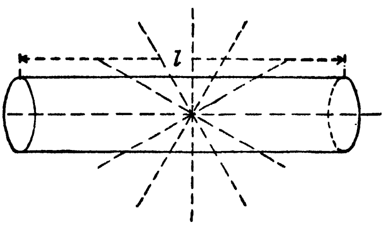


FIG. 57.

$$w = \Phi I' = 4\pi I' \text{ ergs,} \tag{1}$$

for each line of force will have cut the whole current I' once. In moving the distance of 1 cm, each line cuts N/l wires, so that the intensity of the field

$$H = \frac{W}{l} = \Phi I'\frac{N}{l} = \frac{4\pi N}{l}I' \text{ oersteds.} \tag{2}$$

The thickness of the insulation is assumed to be infinitesimal.

If the number of lines penetrating the ends of the solenoid is not negligible but still is small, a correction can be made in the following manner:

The surface area of a sphere circumscribed (Fig. 58) about the solenoid is $4\pi R^2$, and the area of the two segments cut off by the ends of the solenoid is approximately $2\pi r^2$. The number of flux lines that penetrate the loops of wire then is

$$\Phi' = 4\pi\left(1 - \frac{2\pi r^2}{4\pi R^2}\right) = 4\pi\left[1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right].$$

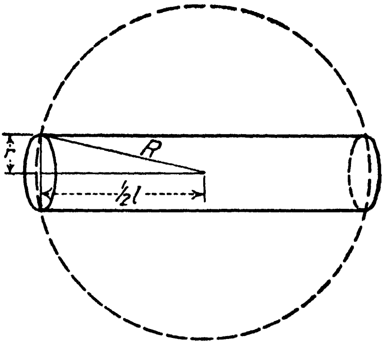


FIG. 58.

The intensity of the field near the center of the solenoid therefore is

$$H = \frac{4\pi N}{l}\left[1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right]I'; \tag{3}$$

and the total number of lines linking the central portion of the solenoid is

$$\Phi = HA = \frac{4\pi NA}{l}\left[1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right]I' = K'I' = \frac{K'}{10}I = KI, \tag{4}$$

whence the constant of the solenoid is

$$K = \frac{2\pi NA}{5l}\left[1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right]. \tag{5}$$

The constant K represents the number of flux lines linking the central section of the solenoid when the current in the solenoid is 1 amp. The area A of the cross section is calculated from the diameter of the solenoid which is measured from center to center of the wires. The diameter of the wire is assumed to be small compared with that of the solenoid.

An equation for the intensity of the magnetic field at any point of the axis of a solenoid, derived by another method, is

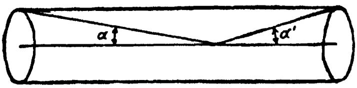


FIG. 59.

$$H = \frac{2\pi NI}{10l}(\cos \alpha + \cos \alpha'),$$

where the angles are those shown in Fig. 59.

The intensity of the field at the center of the solenoid from this equation is

$$H = \frac{2\pi NI}{5l} \cos \alpha,$$

and

$$\Phi = HA = \frac{2\pi NAI}{5l} \cos \alpha = KI,$$

from which

$$K = \frac{2\pi NA}{5l} \cos \alpha. \tag{6}$$

Both of the equations [(5),(6)] are only close approximations. When the coil has more than one layer of wire, the magnitude of K is the sum of the values for the separate layers.

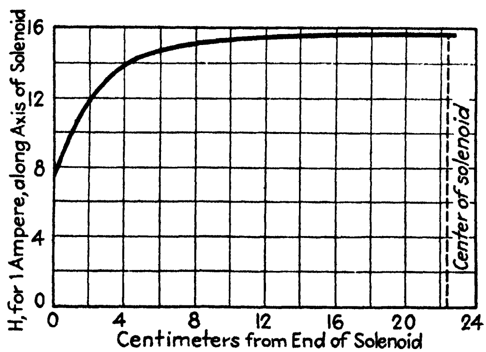


FIG. 60.

The intensity of the magnetic field produced by 1 amp within U. of M. solenoid No. 5, at different points on the axis, from one end of the solenoid to the center, is shown in Fig. 60.

Calculate K for a solenoid by both the equations (5), (6).

EXPERIMENT 43

MUTUAL INDUCTANCE OF A CURRENT INDUCTOR. CALCULATED

Mutual inductance is a constant representing the ratio between the e.m.f. induced in one coil and the rate of change of the current in the other. Then the e.m.f. in the secondary is

$$e = -M\frac{di}{dt} \equiv M\frac{I_P}{t} \text{ volts,} \tag{1}$$

where M is its mutual inductance

Imagine the current in the primary to be changing at the rate of 1 amp/second. Then the mutual inductance M is numerically equal to the induced e.m.f., e. The magnetic flux produced by

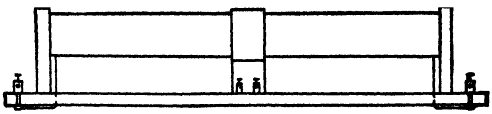


FIG. 61.

1 amp, if the current is diminishing, collapses in 1 second; or, if the current is increasing, is being established. In either case, if the second coil is wound over the central portion of a long solenoid as shown in Fig. 61, all this flux due to 1 amp cuts all the loops of the secondary coil. The number of lines (maxwells) in this flux is represented by the solenoid constant K, which was calculated in Exp. 42. The induced e.m.f. e_1' in each loop of the secondary is numerically equal to the number of lines that cut the loop per second, which in this case is K. The e.m.f. in volts per turn, then, is

$$e_1 = \frac{e_1'}{10^8} = \frac{K}{10^8},$$

and

$$M \equiv N_2 e_1 = \frac{N_2 K}{10^8} \text{ henrys,} \tag{2}$$

where N_2 is the number of turns in the secondary coil.

This expression may also be derived from

$$e = M\frac{I_P}{t} = \frac{N_2 \phi}{10^8 t},$$

from which

$$M = \frac{N_2\phi}{10^8 I_P} = \frac{N_2 K}{10^8}\text{ henrys,} \tag{3}$$

where the constant of the solenoid

$$K = \frac{\phi}{I_P}.$$

Calculate the mutual inductance of a current inductor and compare the computed magnitude with that of the certificate.

EXPERIMENT 44

SELF-INDUCTANCE OF A STANDARD SOLENOID. CALCULATED

Self-inductance is a constant representing the ratio of the counter-e.m.f. induced in a coil or any part of a circuit and the rate at which the current in the same coil or circuit is changing. Then the counter-e.m.f. of self-induction is

$$e_L = -L\frac{di}{dt} \equiv L\frac{I}{t}\text{ volts,} \tag{1}$$

where L = the self-inductance.

Then, as in the case of mutual inductance, the self-inductance L is numerically equal to the counter-e.m.f. e_L when the current in the circuit is changing at the rate of 1 amp/sec. The counter-e.m.f. of self-induction per turn then would be

$$e_1 = \frac{e_1'}{10^8} \cong \frac{K}{10^8},$$

and

$$L \cong N_1 e_1 \cong \frac{N_1 K}{10^8}\text{ henrys.} \tag{2}$$

Figure 60 (Exp. 42) shows that the loops near the ends of a solenoid are not linked by the whole magnetic flux. For this reason, because of the assumption that all the flux which links the central portion of the solenoid cuts all the loops of the solenoid, the foregoing equation [(2)] gives a value for L that is too high.

A good expression[1] for the self-inductance of a solenoid is

$$L = \frac{4\pi^2 N_1^2}{10^9}\left[\frac{2a^4 + a^2 l^2}{\sqrt{4a^2 + l^2}} - \frac{8a^3}{3\pi}\right]\text{ henrys,} \tag{3}$$

[1] *U. S. Bur. Standards Bull.*, Vol. III, p. 303, 1907; Vol. IV, p. 385, 1907–1908.

where a and l are the radius and length of the solenoid and N_1 is the number of turns/cm. The length and radius are measured from center to center of loop wires, and the diameter of the wire is assumed to be small compared to that of the solenoid.

Calculate the magnitude of the self-inductance of the standard solenoid which is part of the current inductor of Exp. 43, using Eq. (3). Compare this value with the approximate value calculated by Eq. (2) and with the certified value.

EXPERIMENT 45

RELATION OF THE QUANTITY OF INDUCED ELECTRICITY IN THE SECONDARY CIRCUIT TO THE RESISTANCE IN THAT CIRCUIT AND TO THE CURRENT CHANGE IN THE PRIMARY

Apparatus.—Mutual-inductance standard M; double-break key K; milliammeter; etc.

Method and Manipulation.—Connect the apparatus as shown in Fig. 62. The resistance $R + M$ in the secondary circuit is adjusted to give critical damping to the galvanometer, and the current in the primary circuit is adjusted to a magnitude such that when the double-break key K is depressed the displaced charge in the secondary produces a large throw (10 to 20 cm).

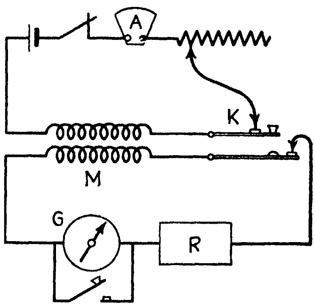


Fig. 62.

It should be noted that the primary circuit is opened first, and then the secondary. The time interval between the two need not exceed 0.0001 seconds when the coils, as is the case with inductance standards, have no iron core.

Throws are taken on opening rather than on closing of the circuit because the galvanometer is more sensitive when its coil moves on open circuit and because the student at this point is not familiar with the determination of the constant on closed circuit and gets valuable experience in eliminating errors due to thermoelectric currents.

When the primary circuit is opened, the collapsing flux of the primary coil induces an e.m.f. in the secondary coil, whose magnitude

$$\bar{e}_M = -M\frac{di}{dt} \equiv M\frac{I_P}{t} = R_S\bar{i}_S \text{ volts,} \tag{1}$$

where the current is assumed to change uniformly from its maximum to zero value. Then

$$MI_P = R_S\bar{i}_S t = R_S Q_S,$$

and

$$Q_S = M\frac{I_P}{R_S}. \tag{2}$$

This equation (2) shows that the quantity induced in the secondary varies directly as the current in the primary and inversely as the total resistance $R_S = R + M + G$ in the secondary. The student should also derive these relationships by direct reasoning without the use of an equation.

Since the galvanometer throws $d \propto Q_S$, it is necessary only to show that

$$d \propto \frac{I_P}{R_S}.$$

1. To show that $d \propto I_P$, take sets of throws from several values of the current differing by approximately equal steps from the lowest to the highest. Tabulate the observations and the corrected throws, and plot the observed relationship between d and I_P.

2. To show that $d \propto 1/R_S$, take sets of throws with the same I_P and several different resistances in the secondary circuit. The lowest resistance r used should be that which gives critical damping. To give a uniform distribution of the throws along the scale, the successive values of the resistances used for R_S should be r, $\frac{4}{3}r$, $2r$, and $4r$, where $r = R + M + G$, in which R is the original resistance in the box when the galvanometer is critically damped.

Tabulate the observations, including the open-circuit null point, the throws corrected for disproportionality, and the several values of $1/R_S$; and plot the relationship between d and $1/R_S$.

In this experiment the discharge through the galvanometer takes place on closed circuit, but the coil moves on open circuit. The throw sensitivity of the galvanometer then is the open-circuit sensitivity, and, since a thermo-e.m.f. almost always is present in the secondary circuit, the galvanometer coil is nearly

always deflected slightly by a thermoelectric current. The null reading on closed circuit then is not that on open circuit. The throws must be measured from the open-circuit null point regardless of whether that is to the right or to the left of the closed-circuit null point. The condition corresponds to that when throws are being taken from either elongation of a slightly vibrating coil (see Exp. 31). *The open-circuit throws are always measured from the open-circuit null point* and never from the slightly deflected position or an elongation. (The null displacement due to a thermoelectric current may be completely eliminated by making one of the connecting wires in the galvanometer circuit a part of another circuit in which the resistance may be adjusted until the potential difference across the wire just balances the thermo-e.m.f.)

EXPERIMENT 46

MUTUAL INDUCTANCE

From Eq. (2) (Exp. 45)

$$M = \frac{Q_S R_S}{I_P}. \tag{1}$$

Choosing one or more favorable points on the plots of Exp. 45, obtain from them the corresponding values of d, R_S, and I_P. The only additional datum required for the determination of M is the figure of merit of the galvanometer on open circuit. This is obtained from the throw produced by the discharge from a standard condenser, as was done in Exp. 32. Then

$$Q_S = Fd_S \text{ coulombs};$$

or, more easily, Q_S may be obtained from

$$\frac{Q_S}{Q} = \frac{Q_S}{CE} = \frac{d_S}{d},$$

where Q and d are the known or observed magnitudes of the condenser discharge and the throw produced by it.

1. Determine the mutual inductance of the mutual-inductance standard of Exp. 45, and compare it with the certified value.

2. Determine the mutual inductance of the current inductor of Exp. 43, and compare the result with the calculated value. This will require additional data for only one magnitude of the throw d

corresponding to one current magnitude I_P. The R_S must be readjusted for the difference in the resistances of the two secondary coils in order that it may have the predetermined magnitude for the desired critical damping.

In measuring mutual inductance it is immaterial which coil is used for the primary. It is customary, however, to use the one having the lower resistance.

EXPERIMENT 47

FIGURE OF MERIT OF A BALLISTIC GALVANOMETER ON CLOSED CIRCUIT

When a ballistic-galvanometer coil is moving on closed circuit, the throw sensitivity is decreased greatly by damping. Since the amount of damping depends on the resistance, it is necessary to determine the figure of merit for each particular circuit at the time of the experiment. It is practical, however, to use repeatedly a predetermined figure of merit provided the same resistance is always employed in the galvanometer circuit and the galvanometer adjustment has not been changed.

In any closed circuit the average magnitude of an instantaneous e.m.f.

$$\bar{e} = M\frac{I_P}{t} = \frac{N_S\phi_S}{10^8 t} = R_S\bar{\imath}_S \text{ volts,} \tag{1}$$

from which

$$MI_P = \frac{N_S\phi_S}{10^8} = R_S\bar{\imath}_S t = R_S Q_S;$$

and the figure of merit

$$F = \frac{Q_S}{d} = \frac{MI_P}{R_S d} = \frac{N_S\phi_S}{10^8 R_S d} \text{ coulombs/cm.} \tag{2}$$

This equation gives the expressions for the figure of merit in the two types of the magnetic standard—one in which the mutual inductance of two fixed coils is known, and the other in which $N\phi$ is given.

1. Standard Mutual Inductance.—With this standard in the galvanometer circuit

$$F_1 = \frac{MI_P}{R_S d_1} \text{ coulombs/cm,} \tag{3}$$

where M is certified; I_P and R_S, measured; and d, observed.

2. Standard Current Inductor.—This is a mutual-inductance standard in which one of the coils is a standard solenoid and the mutual inductance is calculable [see Exps. (42), (43)] and is

$$M = \frac{N_2K}{10^8} \text{ henrys.} \tag{4}$$

Then, substituting this expression for M in Eq. (3), the expression for the figure of merit, determined with the current inductor, becomes

$$F_2 = \frac{MI_P}{R_S d} = \frac{N_2KI_P}{10^8 R_S d_2} \text{ coulombs/cm.} \tag{5}$$

3. Hibbert Magnetic Standard.—This is a very convenient standard and consists of a coil of N turns which can be dropped through a radial field having a known number of flux lines. When this standard is in the galvanometer circuit, dropping the now unnecessary subscripts, Eq. (2) becomes

$$F_3 = \frac{N\phi}{10^8 Rd} \text{ coulombs/cm.} \tag{6}$$

4. Cenco Variable Magnetic Standard.—This standard consists of a coil controlled by two strong spiral springs in a radial magnetic field. The coil can be turned to any desired point where it is held in place until released. It then springs to the null position, producing a change in the magnetic flux linking the coil by an amount that is given on the scale in line turns ($N\phi$). The advantage over the Hibbert magnetic standard is the ability to make the throws from the standard nearly equal to those given by the quantities to be measured. The throws then can be compared directly without the use of a correction curve.

The expression for the figure of merit is the foregoing Eq. (6).

The type of the magnetic standard that uses a permanent magnetic field is the more convenient but has the disadvantage of a possible diminution of the magnetic flux with time and of having a larger temperature coefficient. The temperature coefficient of the mutual-inductance standard is +0.000034 per 1°C.; and that of the permanent-magnet standard, −0.00035 per 1°C.

In all closed-circuit measurements, the question arises regarding the possible effect of the thermoelectric current which usually

is present in the secondary circuit. The throw, therefore, is nearly always taken from a slightly deflected position. Also, the thermoelectric current continues to exert a torque on the coil after the discharge has given the coil its rotational energy and continues to do so until the coil has reached its elongation.

How much do these effects influence the throw, and from what point must the throw be measured?

Let D (Fig. 63) represent the throw when taken from the true null point when there is no thermoelectric current in the circuit. The line BC then represents the torque $T\theta$ of the suspensions at elongation, and the triangle ABC that part of the kinetic energy of the coil which is transformed into the potential energy of the twisted suspensions.

When a thermoelectric current is deflecting the coil the distance d, it is exerting a torque $T\phi$ on the coil equal to that of the twisted suspensions. The same discharge now gives the coil the same kinetic energy that it gave it at the true null point; but as the coil turns, the torque $T\phi$ continues to act and imparts additional energy to the turning coil. When finally the coil reaches its elongation, the shaded area A_1B_2 represents, in arbitrary units, the energy given the coil by the thermoelectric current; and the triangle $A_1B_1C_1$, the part of the original kinetic energy of the coil which has been transformed into the potential energy of the twisted suspensions. Since the kinetic energy given the coil by the discharge is that in the first case, this energy will have been exhausted when the same part of it has been transferred to the suspensions. Then $\Delta A_1B_1C_1 = \Delta ABC$, and $A_1B_1 = AB = D$. The throw when taken from a deflected position then is measured from its deflected position. The thermoelectric current, then, changes only the point from which the throws are measured. The fact that the coil at null reading hangs in a slightly different part of the radial field can change only the figure of merit, the amount of this change ordinarily being inappreciable.

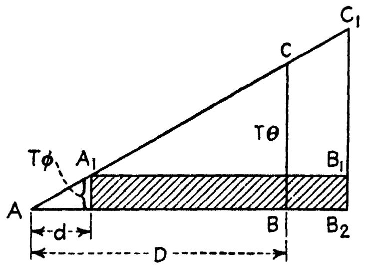


FIG. 63.

In a similar manner it can be shown that if the throw and the thermoelectric deflection are on opposite sides of the open-circuit null point, the throws also are measured from the deflected position.

The following statement can now be made: *All ballistic throws must begin with the coil at rest; but throws moving on open circuit are measured from the open-circuit zero, and throws moving on closed circuit are measured from the closed-circuit zero if they originated on closed circuit.* Therefore, when the galvanometer throws are taken on closed circuit, with the circuit remaining

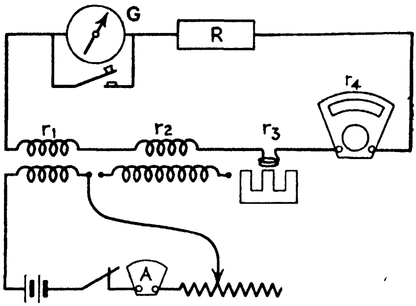


Fig. 64.

closed throughout, no attention need be given to the thermoelectric current.

Determine the figure of merit of a ballistic galvanometer by means of each of the four standards when it is in circuit with the critical-damping resistance (Exp. 33), which is

$$R + r_1 + r_2 + r_3 + r_4,$$

when all four standards are in circuit at the same time, as shown in Fig. 64. The battery circuit is shown connected to the primary of the mutual-inductance standard ready for an observation. When the observations are taken with the current inductor, either coil may be used as the primary just as with any mutual-inductance standard. In case the four standards are not in circuit at the same time, care must be taken to compensate in R for any difference in the resistance of the substituted standards.

Tabulate the four determined values of the figure of merit, and calculate the ratio between the average value and that obtained for the same galvanometer on open circuit, in Exp. 32.

It follows from Eqs. (2), (3), (5) that

$$N_S\phi_S = 10^8 Q_S R_S. \tag{7}$$

$$10^8 M I_P = 10^8 Q_S R_S. \tag{8}$$

$$N_2 K I_P = 10^8 Q_S R_S. \tag{9}$$

Then, when a flux change ϕ_X to be measured has taken place in a coil of N_X turns in the same circuit,

$$N_x\phi_x = 10^8 Q_x R_S. \tag{10}$$

It then follows that $N_x\phi_x$ can be compared with the known constants of either of the four standards. Dropping the subscripts *s* for the standards,

$$\frac{N_x\phi_x}{N\phi} = \frac{10^8 Q_x R_S}{10^8 Q R_S} = \frac{Q_x}{Q} = \frac{d_x}{d}. \tag{11}$$

Similarly,

$$\frac{N_x\phi_x}{10^8 M I_P} = \frac{d_x}{d}, \tag{12}$$

and

$$\frac{N_x\phi_x}{N_2 K I_P} = \frac{d_x}{d}. \tag{13}$$

The measurement of the change in line turns, then, is usually made by comparison with one of the four standards, using the applicable one of the foregoing three equations.

EXPERIMENT 48

FIGURE OF MERIT ON OPEN CIRCUIT FROM A CLOSED-CIRCUIT STANDARD AND ON CLOSED CIRCUIT FROM AN OPEN-CIRCUIT STANDARD

Apparatus.—Mutual-inductance standard; standard mica condenser; condenser discharge key; double-break key; etc.

1. Open-circuit Figure of Merit from Mutual-inductance Standard.—Connect the apparatus as shown in Fig. 62 (Exp. 45). The mutual-inductance standard, when the current in its primary changes, displaces a known quantity Q in the closed galvanometer circuit; but the galvanometer circuit is opened before the coil turns an appreciable distance. The coil then turns on open

circuit, and the throw is measured from the open-circuit null point. The figure of merit on open circuit [Eq. (3), Exp. 47] is

$$F_1 = \frac{MI_P}{R_S d_1} \text{ coulombs/cm.} \tag{1}$$

Compare the value obtained with that determined by means of a standard condenser.

2. Closed-circuit Figure of Merit from a Standard Condenser. Connect the apparatus as shown in Fig. 65. The parts are connected in the standard manner except that a lug A under the lowest spring is connected to B, so that, after the condenser discharges and before the coil turns appreciably, the galvanometer circuit is closed. Then the figure of merit on the particular closed circuit is

$$F_2 = \frac{CE}{d_2} \text{ coulombs/cm,} \tag{2}$$

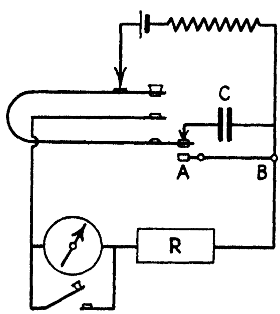


FIG. 65.

where C is the capacitance of the condenser in farads.

The key must be held down to keep the circuit closed until the throw reading is taken. The resistance R should be that which gives critical damping so that the result may be compared with that obtained by Exp. 47. The time interval between the condenser discharge and the short-circuiting should be as short as possible to reduce the undamped part of the coil's motion. When the discharge key has two binding posts for the attachment of each instrument, short-circuiting the two binding posts marked "App" makes the connection AB of Fig. 65. If a thermo-e.m.f. is present in the galvanometer circuit, the question again arises, From what point should the throw be measured?[1] Since the correction to be made depends on the amount of damping, it is better in this case to eliminate the thermo-e.m.f. as suggested in Exp. 45; but a correction can be made from the curve of Fig. 66. After an observed throw, the coil is allowed to swing beyond the null point on the closed circuit. This distance may be zero, but the resistance must not be less than that which makes the gal-

[1] The complete discussion of this matter is given in *Phys. Rev.*, Vol. 23, p. 414, 1906; n.s. 7, p. 641, 1916.

vanometer critically damped. The distance that the coil turns beyond the null point gives the necessary datum for obtaining the correction from the curve and the observed thermoelectric deflection on the closed circuit. This correction is either added to or subtracted from the observed throw as measured from the open-circuit zero. Thus if the thermoelectric deflection is 0.25 cm in the same direction as the throw, and the galvanometer

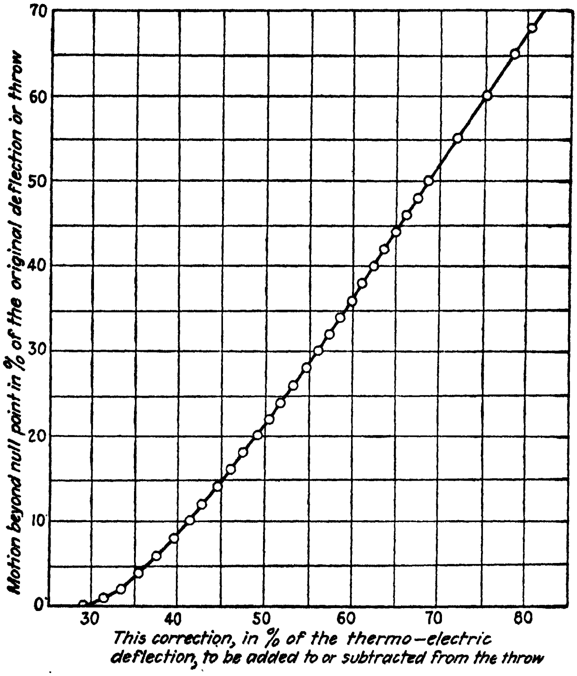


FIG. 66.

is critically damped, the curve shows that the observed throw is too large by 28 per cent of 0.25, or 0.07 cm. The correction to the throw then is -0.07 cm.

It is not recommended that either of the methods of this experiment be used when the proper standards are available for the particular kind of circuit used, but they give the student valuable experience with the various conditions that affect the throw of a ballistic galvanometer. The results procured by these methods should be compared with those obtained in Exps. 32, 48, in order

to show what conformity can be attained. This experiment should be omitted if time does not permit the performance of all the remaining experiments of the course.

EXPERIMENT 49

FLUX DENSITY OF A MAGNETIC FIELD AND THE POLE STRENGTH OF A BAR MAGNET

1. Repeat Exp. 19, Chap. VI; but now measure the flux density, using the jerk-type exploring coil, with five different magnetizing currents of 0.25, 0.50, 1, 2, and 5 amp, in consecutive order from the lowest to the highest; and then in the reverse order. Plot the flux densities against the magnetizing currents.

The *snap-type exploring coil* rotates through 180 deg and therefore cuts the linking flux twice. It has the disadvantage of requiring more space. The *bismuth-spiral type* depends on the resistance of bismuth changing with flux density. The resistance is measured in the ordinary manner within and out of the field, from which the flux density is obtained from a calibration curve. It is used for measuring very high flux densities only.

2. Add to or substitute for the exploring coil in the foregoing galvanometer circuit a coil of from 20 to 40 turns, which fits closely over a magnet whose pole strength is to be measured. Center the magnet within the coil. Then, in jerking the coil off, the coil cuts $4\pi m$ maxwells. The observed throws are compared with those obtained with the magnetic standard, whence

$$\frac{N_x\phi_x}{N\phi} = \frac{4\pi m N_x}{N\phi} = \frac{d_x}{d},$$

from which the pole strength

$$m = \frac{N\phi}{4\pi N_x} \cdot \frac{d_x}{d} \text{ units.}$$

EXPERIMENT 50

HORIZONTAL INTENSITY, INCLINATION, AND TOTAL INTENSITY OF THE EARTH'S MAGNETIC FIELD. EARTH INDUCTOR METHOD

1. **Measurement of H.**—The earth inductor is a large exploring coil of the snap type and consists of a coil of wire which can be

rotated through 180 deg in the earth's magnetic field. In Fig. 67 the coil is wound on the circular frame which rotates within the square frame which can be turned into any position and clamped. The circular frame rotates on an axis perpendicular to the axis of the square one, as shown.

The square frame is clamped in a vertical position to the supporting frame, and then by the aid of a magnetic needle its plane is placed perpendicular to the magnetic meridian. The coil

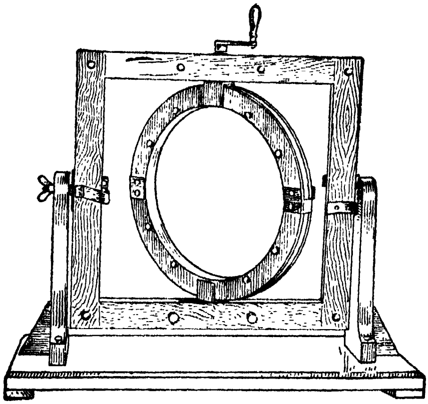


FIG. 67.

is then connected in series with a ballistic galvanometer, a magnetic standard, and the critical damping resistance.

The circular frame holding the coil is turned into the plane of the square frame and then is rotated quickly through 180 deg. The quantity of induced charge is obtained from the throw by comparison with that from a magnetic standard.

The number of flux lines linking the coil in its initial position is AH, where A is the average area of the N_x loops in the coil and H is the horizontal component of the earth's magnetic field. In turning through 180 deg, these lines are cut twice, so that the total number of lines cut is $2AH$.

When the Cenco or the Hibbert standard is used for comparison,

$$\frac{2AHN_x}{\Phi N} = \frac{d_x}{d},$$

from which

$$H = \frac{\Phi N}{2AN_x}\frac{d_x}{d} \text{ oersteds.} \tag{1}$$

2. Measurement of V.—The square frame is now turned into a horizontal position and clamped, and throws are obtained in the same manner as in the measurement of H. The magnetic flux now linking the coil is AV maxwells, where V is the vertical component of the earth's magnetic field. Then, as in the case of H,

$$V = \frac{\phi N}{2AN_x} \frac{d_x}{d} \text{ oersteds.} \tag{2}$$

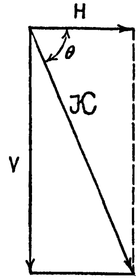


Fig. 68.

3. Measurement of Inclination.—With reference to Fig. 68, the angle of inclination is

$$\theta = \tan^{-1}\left(\frac{V}{H}\right); \tag{3}$$

or directly from

$$\frac{d_V}{d_H} = \frac{V}{H} = \tan\theta. \tag{4}$$

4. The Total Intensity of the Earth's Magnetic Field.—The total intensity $\mathcal{H}$ is obtained from

$$\mathcal{H} = \frac{H}{\cos\theta} \text{ oersteds.} \tag{5}$$

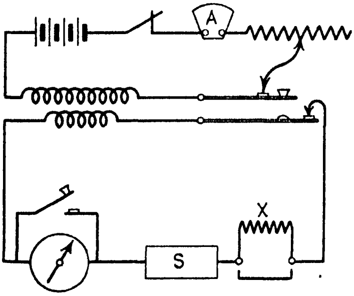


Fig. 69.

EXPERIMENT 51

ABSOLUTE DETERMINATION OF RESISTANCE. CURRENT-INDUCTOR METHOD

Apparatus.—The current inductor of Exp. 43, 47; double-break key; an "unknown" resistance of 400 ohms to be measured and a means for easily short-circuiting or of withdrawing it from the galvanometer circuit.

Method of Manipulation.—The apparatus is connected as shown in Fig. 69, where X is the unknown resistance to be measured. The resistance inserted into S is that which gives critical damping when the resistance X is not in circuit. Throws are first taken with X out of the circuit; then from Exp. 47 Eq. (9),

$$N_2 K I_P = 10^8 Q_S R_S = 10^8 R_S F d_1, \tag{1}$$

from which the total resistance in the galvanometer circuit is

$$R = \frac{N_2KI_1}{10^8Fd_1}, \tag{2}$$

where I_1 is the I_P in Eq. (1) and F is the figure of merit of the galvanometer on open circuit. The throw d_1 is measured from the open-circuit null point, and K was calculated in Exp. 42. To be an absolute method the figure of merit must be calculated from the constant in Exp. 30; but since that, as determined by the student, is liable to have a comparatively large error, the figure of merit should be obtained from the throw of a standard condenser.

Throws are now taken with the unknown resistance X in circuit. Then the total resistance in the circuit is

$$R + X = \frac{N_2KI_2}{10^8Fd_2}. \tag{3}$$

The difference between the two resistances $(R + X)$ and R of Eqs. (3), (2) gives the value of the unknown resistance introduced into the circuit. This is

$$X = \frac{KN_2I_2}{10^8Fd_2} - \frac{KN_2I_1}{10^8Fd_1} = \frac{KN_2}{10^8F}\left(\frac{I_2}{d_2} - \frac{I_1}{d_1}\right)\text{ohms.} \tag{4}$$

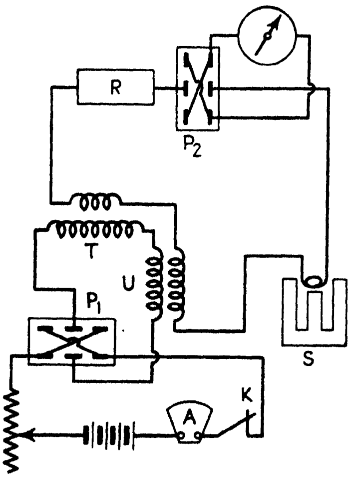


FIG. 70.

EXPERIMENT 52

MAGNETIC PROPERTIES OF IRON. J, B, κ, μ, $\mathfrak{R}$, AND HYSTERESIS

Apparatus.—In Fig. 70, T is a current inductor whose solenoid is to be employed for magnetizing the specimen of iron under test. The number of turns on both its coils is known. The solenoid is placed at right angles to the magnetic meridian to avoid, as far as is practical, the magnetizing action of the earth's magnetic field on the iron. The two coils U consist of a fixed primary and an adjustable secondary. This secondary is in the galvanometer

circuit with the fixed secondary of the magnetizing solenoid of current inductor T. The secondaries are so connected that the e.m.fs. induced in them oppose each other. The secondary of coils U is adjusted until the largest current (5 amp) to be employed produces, on the opening of the circuit, no appreciable throw on the galvanometer when the iron is not within the magnetizing solenoid. All throws then when the iron is in the solenoid are produced by a change of magnetic flux in the iron alone. The resistance in R is such as to give critical damping in conjunction with the other resistances in the circuit. The magnetic standard is represented by S. The commutator P_1 in the primary circuit enables the current, and with it the magnetic field within the magnetizing solenoid T, to be reversed. Since decreasing magnetization of the iron produces throws in the galvanometer that are reverse in direction to those produced by increasing magnetization, and because galvanometer throws should always be taken in the same direction, it is necessary to employ the commutator P_2 to reverse its connections to the galvanometer whenever the induced current changes direction. This reversal takes place when a change is made from an increasing to a decreasing magnetizing current at the two points in the cycle where this current has its maximum value. It is well to remember that when one commutator is reversed, the other is not. One is reversed when the magnetizing current has its maximum value; and the other, when the current is zero. The iron to be magnetized is in the form of a long, thin wire, somewhat shorter than the magnetizing solenoid.

Method and Manipulation.—Demagnetize the iron wire in a demagnetizing solenoid provided for the purpose. Then, always keeping the iron perpendicular to the earth's magnetic field, insert the iron wire into the unenergized solenoid with the aid of a copper wire hard soldered to its end. To test for the presence of residual magnetization in the iron, jerk the iron out of the solenoid. This withdrawal should produce no throw on the galvanometer. If it does, try to remove the magnetism from the iron; but, if finally any remains, take the initial magnetism into consideration as a part of the total magnetism that will be induced in the iron. Care must be taken to make the correction in the proper sense.

With the iron within the magnetizing solenoid T, increase the current by small steps at first until finally a considerable increase produces but little effect upon the galvanometer; then, after reversing the commutator P_2, decrease the current to zero by the same steps. Then reverse the commutator P_1, and repeat the process, reversing P_2 at the proper time; and then again reverse P_1, and increase the current to the maximum. The ballistic throw and the ammeter reading are taken at each change. These give the data for calculating the changes in the magnetizing field and in the induction. The sign of the throws changes whenever the commutator P_2 is reversed. The actual throws obtained are all in the same direction on the scale.

Before taking the final set of readings, it is desirable to make a preliminary run through the whole cycle and mark the points on the rheostats to which successive adjustments are to be made for obtaining a desirable set of throws. The magnetizing-current values may be 0.2, 0.5, 1, 2, 3, and 5 amp.

It must be remembered that the iron must not be removed from the solenoid during the experiment, that each successive current change must be made as quickly as is practicable, and that after a current change is made it must not be altered before the throw reading for that change is taken.

Obtain the comparison throws from the magnetic standard.

1. Tabulate the magnetizing currents, the observed throws, the corrected throws (using the correction curve), and the algebraic sum of the corrected throws including the residual throw.

2. *Hysteresis Curve.*—The flux due to the iron is proportional to the algebraic sum of the throws.

$$\Phi_x N_x = 10^8 Q_x R = 10^8 RFd_x;$$

$$\Phi_x = \frac{10^8 RF}{N_x} d_x = Kd_x. \tag{1}$$

The magnetizing field is proportional to the magnetizing current. The magnetization and the hysteresis curves can therefore be obtained by plotting the magnetizing current on the axis of abscissas and the algebraic sum of the throws on the axis of ordinates.

3. Mark with a cross a point near the knee of the magnetization curve. For this one point determine the following quantities:

H—Intensity of magnetizing field.

$$H = \frac{4\pi NI}{10l}\left[1 - \frac{r^2}{2\left(\frac{l^2}{4} + r^2\right)}\right]. \tag{2}$$

Since the radius of the magnetizing solenoid is small, the correction for the ends may be neglected.

Φ_x—*The total amount of magnetic flux* in the iron at the given point is determined by comparing the sum of the corrected throws for that point with the corrected throw obtained from the standard.

$$\frac{\Phi_x N_x}{\Phi N} = \frac{d_x}{d}; \qquad \Phi_x = \frac{\Phi N}{N_x}\frac{d_x}{d}. \tag{3}$$

J—Intensity of magnetization.

$$J = \frac{M}{V} = \frac{ml'}{sl} \cong \frac{4\pi m}{4\pi s} = \frac{\Phi}{4\pi s}. \tag{4}$$

where Φ is the number of flux lines in the iron, and s the cross section of the iron in square centimeters. The number of induced lines per square centimeter is

$$\frac{\Phi}{s} = 4\pi J. \tag{5}$$

B—Magnetic induction or flux density refers to the total number of flux lines per square centimeter within the iron. It includes the lines of the magnetizing field and the lines induced in the iron. Then

$$B = B_H + 4\pi J. \tag{6}$$

κ—*Magnetic susceptibility* is the ratio of the intensity of magnetization to the intensity of the magnetizing field.

$$k = \frac{J}{H}. \tag{7}$$

μ—*Magnetic permeability* is the ratio of the magnetic flux density to the intensity of the magnetizing field.

$$\mu = \frac{B}{H}. \tag{8}$$

$\mathcal{R}$—*Magnetic reluctance*

$$\mathcal{R} = \frac{l}{\mu s}.$$

The reluctance is that of the iron and the air which the flux lines penetrate. This reluctance affects the magnitudes of nearly all the foregoing listed quantities. But when the wire is long compared to its diameter, these values are nearly those obtained with a ring.

EXPERIMENT 53

SELF-INDUCTANCE. MODIFICATION OF MAXWELL'S METHOD

Apparatus.—One of the coils of the mutual-inductance standard of Exps. 46, 47 whose self-inductance is to be measured; milliammeter; ballistic galvanometer; double-break key; slide-wire resistance; etc.

Method and Manipulation.—Connect the apparatus to form a Wheatstone bridge as shown in Fig. 71, placing the slide-wire resistance and a resistance box, adjustable to 0.1 ohms, into the arm of the bridge which contains the coil whose self-inductance L is to be measured. The battery circuit usually requires no rheostat, but that must be inserted if it is necessary. The double-break key opens the battery circuit and the galvanometer branch in succession. One of the galvanometer leads contains a high resistance $H.R.$ spanned by a shunting switch. This high resistance is introduced into the galvanometer branch to lessen the potential-difference sensitivity of the galvanometer when the first rough adjustment is being made and is shunted out for the final balancing of the bridge.

The bridge responds to slight changes in the temperature of the copper coil, so that it is often necessary to keep the slide on the slide-wire resistance in continuous motion. This heating effect may be reduced by diminishing the current, but often it is difficult to bring the coil to rest at the exact closed-circuit null point. In that case the throw is taken whenever the coil is brought to rest at any point within a millimeter of the null point. The small lack of balance is immaterial; the throw in any case is measured from the open-circuit null point.

On opening the circuit, the magnetic flux linking the coil L collapses and thereby induces an instantaneous e.m.f. in the coil which displaces the free electrons throughout the circuit. The whole quantity Q of the discharge passes through the resistances

R_1 and R_3 but divides in passing through the resistances G and $R_2 + R_4$. The induced e.m.f. in the coil is

$$\bar{e} = -L\frac{di}{dt} \equiv L\frac{I_1}{t} = R\bar{\imath},$$

from which

$$L = \frac{RQ}{I_1} \text{ henrys,} \tag{1}$$

where R is the resistance in the circuit, calculated for the condition where the e.m.f. is in the arm R_1, Q the total quantity of charge displaced, and I_1 the current in the coil L before the circuit is

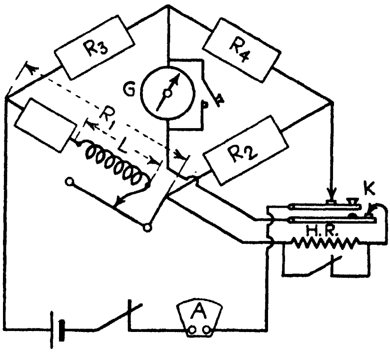


Fig. 71.

opened. The magnitudes of these three quantities are obtained as follows:

1. The current I in the ammeter divides into two parts, I_1 and $I - I_1$, making

$$\frac{I_1}{I - I_1} = \frac{R_3 + R_4}{R_1 + R_2},$$

from which the current in the arm R_1 and therefore the coil is

$$I_1 = \frac{(R_3 + R_4)I}{R_1 + R_2 + R_3 + R_4} \text{ amp.} \tag{2}$$

2. The resistance in the part series and part parallel circuit is

$$R = R_1 + R_3 + \frac{(R_2 + R_4)G}{R_2 + R_4 + G} \text{ ohms,} \tag{3}$$

where R_1 is the total resistance in the arm of the circuit containing the coil whose self-inductance is to be measured.

3. The quantity q discharged through the galvanometer is measured by comparison with a discharge from a standard condenser or from the known open-circuit figure of merit. The throws must be measured from the open-circuit null point as

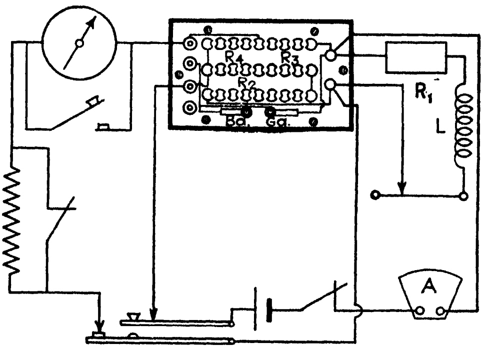


FIG. 72.

explained for such a measurement in Exp. 46. The total quantity Q discharged through the circuit divides into two parts q and $Q - q$, so that

$$\frac{q}{Q-q} = \frac{R_2 + R_4}{G},$$

from which

$$Q = \frac{(R_2 + R_4 + G)q}{R_2 + R_4} \text{ coulombs.} \tag{4}$$

The resistance G of the galvanometer must be known or measured. The self-inductance of the galvanometer coil does not affect the amount of the discharge through the coil, because the influence of the growing magnetic flux during the time of the increase in the current is balanced by the equal reverse effect during the decrease.

The three arms of a plug-type Wheatstone bridge may be used for the three resistances R_1, R_3, R_4. The connections are then made as shown in Fig. 72.

It is convenient to make $R_3 = R_4 = 100$ and R_2 only slightly larger than the resistance of the coil L in the arm R_1. The resistance R_1 is made as nearly equal to R_2 as is convenient before a

test for balance is attempted. If this can be done with some precision, the high resistance HR and its shunt may be omitted.

EXPERIMENT 54

SELF-INDUCTANCE AND CAPACITANCE. ANDERSON'S METHOD

Apparatus.—The adjusted apparatus of Exp. 53; mica condenser C; resistance box r adjustable to 0.1 ohm; rotating commutator.

Method and Manipulation.—This method of measuring the self-inductance of coils (without iron cores) has an advantage over

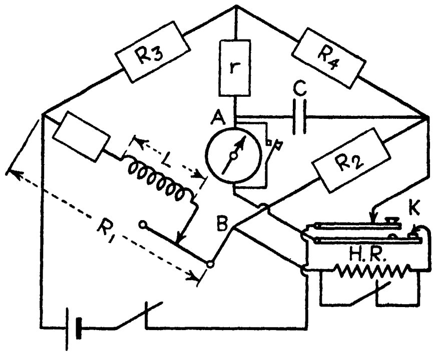


Fig. 73.

that of the foregoing experiment in that it is a null method and does not require the measurement of the current, resistance of the galvanometer, and a quantity of electricity. It also can be made much more sensitive by closing and opening the circuits in continuous succession by means of a rotating commutator.

The disposition of the apparatus is shown in Fig. 73, which is seen to be that of Fig. 71, except that a standard mica condenser C and a resistance box r are added, and the milliammeter is discarded.

When the bridge is adjusted for balance, the condenser C becomes charged. And when the circuits are opened in succession by means of the double-break key K, part of the condenser discharge passes through the galvanometer in a direction opposite that of the charge forced through it by the induction in the coil L. Inspection shows that when the resistance in r is increased, for

example, the part of the condenser discharge that is forced through the galvanometer is increased, while that part of the induced charge which is forced through in the reverse direction is decreased. The resistance r then can be adjusted until these oppositely directed discharges are equal. This condition exists when no throw is observed on opening or closing the circuits. The experiment then consists in first balancing the bridge circuit and then adjusting the resistance r for the throw balance.

Although the two discharges may actually take place in opposite directions through the galvanometer, the effect of them together is that of no discharge. Therefore the result is that which would exist if the equal oppositely directed discharges were exactly simultaneous, and changed in the same manner. There would then be no potential difference across the galvanometer terminals, and no discharge through the galvanometer. The potentials at the points A and B then are equal. The same condition exists, except in the reverse direction, when the current in the circuit is being established and the condenser is being charged.

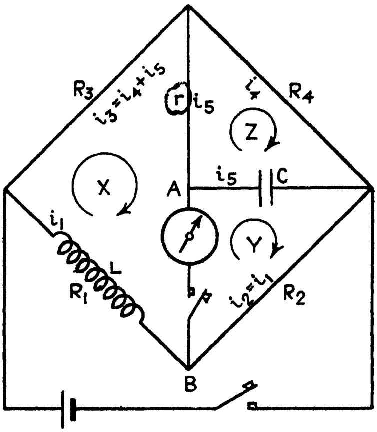


FIG. 74.

Assume the bridge to be balanced for both steady and intermittent currents. Then, referring to Fig. 74, where i with the appropriate subscript represents the instantaneous magnitudes of the current in the various branches,

1. From the balanced Wheatstone net

$$R_1 = \frac{R_2}{R_4}R_3. \tag{1}$$

2. In the mesh X, since the potential at A is that at B, the relationship between the potential differences is

$$R_1i_1 + L\frac{di_1}{dt} = R_3i_3 + ri_5. \tag{2}$$

3. In mesh Y, the current i_5 flowing into or out of the condenser is that in r, and the current in R_2 is that in R_1. Then

$$R_2 i_1 = \frac{q}{C} = \frac{1}{C}\int i_5 dt,$$

from which

$$\frac{di_1}{dt} = \frac{i_5}{R_2 C}. \tag{3}$$

4. Substituting Eqs. (1), (3) in Eq. (2) and transposing,

$$L = CR_2\left[r + R_3\left(\frac{\dot{i}_3}{\dot{i}_5}\right) - \frac{R_2R_3}{R_4}\left(\frac{\dot{i}_1}{\dot{i}_5}\right)\right]. \tag{4}$$

5. From mesh YZ, since the potential at A is that at B,

$$R_2 i_1 = R_4 i_4 - r i_5,$$

from which

$$R_2\left(\frac{\dot{i}_1}{\dot{i}_5}\right) = R_4\left(\frac{\dot{i}_4}{\dot{i}_5}\right) - r. \tag{5a}$$

But

$$\dot{i}_4 = \dot{i}_3 - \dot{i}_5,$$

whence

$$\frac{\dot{i}_4}{\dot{i}_5} = \frac{\dot{i}_3}{\dot{i}_5} - 1. \tag{5b}$$

Then, substituting (5b) in Eq. (5a) and dividing by R_2,

$$\frac{\dot{i}_1}{\dot{i}_5} = \frac{R_4}{R_2}\left(\frac{\dot{i}_3}{\dot{i}_5} - 1\right) - \frac{r}{R_2}. \tag{5c}$$

6. Substituting (5c) in Eq. (4), canceling, and uniting,

$$L = CR_2\left[r\left(1 + \frac{R_3}{R_4}\right) + R_3\right] \text{ henrys,} \tag{6}$$

or

$$L = C[r(R_1 + R_2) + R_2R_3] \text{ henrys.}$$

7. Equation (6) may be employed also for measuring capacitance in terms of a standard self-inductance. Transposing,

$$C = \frac{L}{r(R_1 + R_2) + R_2R_3} \text{ farads.} \tag{7}$$

The balancing of the bridge for continuous currents is made in the same manner as in Exp. 53. But, in place of observing the throw on opening the circuits, the resistance in r is adjusted until there is no throw. If a thermoelectric current is affecting the galvanometer, it should be eliminated by a potentiometer method described in Exp. 45; but if the "balance" for the continuous current is made to the open-circuit null point, the error introduced is small or negligible, and a difficulty in obtaining a balance for the intermittent-current pulses is avoided.

The sensitivity for the intermittent-current balance is increased about one hundred-fold by means of a double rotating commutator. This consists of two commutators mounted on an extension of the shaft of a small motor. Figure 75 shows the construction of the commutators but does not show them in their natural position on the shaft. One brush on each commutator makes contact successively with four segments, two of which are connected to a ring which makes a continuous contact with a second brush. Figure 75 shows each of two brushes making a contact with one of the two blind segments, and therefore they are shown in a position where both circuits are open.

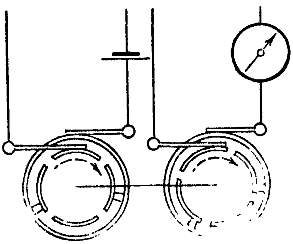


FIG. 75.

The two commutators are clamped to the shaft in such relative angular positions that the galvanometer circuit is always closed at the time when the battery circuit is opened and is open before the battery circuit is closed. In this manner only the unidirectional pulses produced at the opening of the battery circuit can affect the galvanometer. Because the pulses come in rapid succession, they produce a continuous galvanometer deflection.

When the rotating commutator is used, it displaces the double-break key K in Fig. 73. The adjustment for the direct-current bridge balance is made with the motor at rest and the shaft turned into a position where both commutators are closing their respective circuits. For precision work thermoelectric currents should be eliminated. Their effect is relatively small because of the large current equivalent of the many pulses. The galvanometer deflections sometimes are erratic owing to electrostatic

discharges. The commutator segments always should be covered with a thin layer of vaseline, which minimizes this effect, but it may be necessary to ground a point on one of the commutators.

Measure the self-inductance of the coil used in Exp. 53, first using the double-break key, and then using the rotating commutator.

EXPERIMENT 55

SELF-INDUCTANCE. COMPARISON METHOD

Apparatus.—The apparatus of Exp. 54, except that a variable self-inductance standard is substituted for the resistance R_2 as shown in Fig. 76, and the condenser C and resistance r are removed; rotating commutator.

The variable self-inductance standard consists of two coils connected in series, one of them fixed, and the other movable about a vertical axis. The self-inductance of the two coils

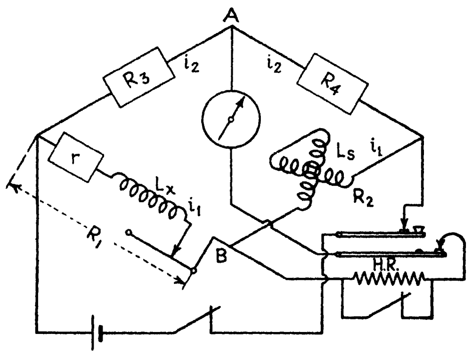


Fig. 76.

depends on their relative position, which can be changed by turning one of the coils through any desired distance. The self-inductance is changed without altering the resistance, so that the balance made for the steady current remains intact. A graduated scale indicates the inductance in millihenrys for any position of the movable coil.

Method and Manipulation.—Balance is obtained with a steady current and then with an intermittent current, as in Exp. 54, except that the balance for the intermittent current is secured by

adjusting the self-inductance of the variable standard in the arm R_2. If the standard is adjustable through only a short range, the values of the resistances R_3, R_4 must be so chosen that a balance may be obtained with some value of the known inductance which is within the range of the standard. The appropriate trial values of the resistances can be calculated from

$$\frac{L_S}{L_x} = \frac{R_4}{R_3},$$

if the approximate magnitude of L_x is known. The resistance box r and the slide-wire resistance must be placed into that arm which requires an additional resistance to make

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}.$$

The balance with a continuous current gives

$$\frac{R_1}{R_2} = \frac{R_3}{R_4},$$

from which

$$R_1R_4 = R_2R_3. \tag{1}$$

In the case of the intermittent current, let i_1 be the instantaneous value of the current in R_1 and R_2, and i_2 that in R_3 and R_4, at the instant t after closing the circuit. At the intermittent-current balance, the potential at A must be that at B. Hence the relationship between the potential differences is

$$R_1i_1 + L_x\frac{di_1}{dt} = R_3i_2, \tag{2}$$

and

$$R_2i_1 + L_s\frac{di_1}{dt} = R_4i_2. \tag{3}$$

From Eqs. (2) and (3)

$$R_1R_4i_1i_2 + L_xR_4\frac{di_1}{dt}i_2 = R_2R_3i_1i_2 + L_sR_3\frac{di_1}{dt}i_2. \tag{4}$$

Substituting R_1R_4 for R_2R_3 [Eq. (1)] in Eq. (4) and canceling,

$$L_xR_4 = L_sR_3,$$

and

$$L_x = \frac{R_3}{R_4} L_s. \tag{5}$$

As with Anderson's method, the sensitivity is greatly increased by substituting a double rotating commutator for the double-break key.

Measure the self-inductance of the coil used in the foregoing experiments, using first the double-break key and then the rotating commutator. Tabulate the five values of L_x, including the probable error obtained for the same coil in Exps. 53 to 55.

APPENDIX

1. LENGTH

1 cm	= 0.39370 in.	1 micron (μ)	= 0.001 mm
1 m	= 3.2809 ft	1 millimicron ($\mu\mu$ or mμ)	= 0.001 μ
1 km	= 0.62137 mile	1 Ångström unit (Å.)	= 10^{-8} cm = 0.1 $\mu\mu$
1 mil	= 0.001 in.		
1 in.	= 2.54 cm		
1 ft.	= 30.46 cm		
1 mile	= 1.609 km		

2. AREA

1 sq cm	= 0.155 sq in.	1 sq in.	= 6.4516 sq cm
1 sq m	= 10.764 sq ft	1 sq ft	= 929.03 sq cm
1 cir cm	= 0.1217 sq in.	1 sq mile	= 2.59 sq km
Area of circle	= $\left(\frac{\pi}{4}d^2\right) = 0.7854d^2$	1 cir in.	= 5.0671 sq cm
1 cir mil	= 0.000001 sq in.	1 cir mil	= 0.00050671 sq mm

3. VOLUME

1 cc	= 0.061 cu in.	1 cu in.	= 16.387 cc
1 cu m	= 35.315 cu ft	1 cu ft	= 0.028317 cu m
1 l	= 61.023 cu in.	1 cu ft	= 28.317 l
1 l	= 2.11336 pt (liquid)	1 pt (liquid)	= 0.47318 l

4. MASS

1-g	= 15.432 gr	1 oz (av.)	= 28.35-g
1 kg	= 2.205 lb	1 lb (av.)	= 453.59-g

5. CIRCULAR MEASURE

1 rad	= 57.296 deg	1 deg	= 0.017453 rad
π	= 3.1416	log π	= 0.49714987
$\frac{1}{\pi}$	= 0.31831		
π^2	= 9.8696		
$\sqrt{\pi}$	= 1.77245		

6. FORCE

1 dyne	= 0.00007233 poundals	1 poundal	= 13825 dynes
1 dyne	= 0.00102-g	1 poundal	= 0.03108 lb
1 dyne	= 22.48×10^{-7} pounds	1 poundal	= 14.10-g
1 megadyne	= 1000000 dynes		

7. PRESSURE

1 atmosphere (standard)	= 76.0 cm of Hg	1 atmosphere (76 cm) of Hg	= 29.92 in. of Hg
1 atmosphere	= 1.0333 kg/sq cm	1 atmosphere	= 14.697 lb/sq in.
1 atmosphere	= 1.0133 megadynes/sq cm	1 atmosphere	= 33.90 ft of H_2O
1 dyne/sq cm	= 0.0004666 poundals /sq in.	1 poundal/sq in.	= 2142.95 dynes/sq cm
1-g wt./sq cm	= 0.014223 lb/sq in.	1 lb/sq in.	= 70.31-g/sq cm
1 cm of Hg (0°C.)	= 13.596-g wt./sq cm	1 in. of Hg (0°C.)	= 34.533-g wt./sq cm
1 cm of Hg (0°C.)	= 0.19338 lb wt./sq in.	1 in. of Hg (0°C).	= 0.4912 lb wt./sq in.
1 megabarye	= 1 megadyne/sq cm		
1 megabaye	= 75.0068 cm of Hg		

8. ENERGY

1 erg	$= 2.373 \times 10^{-6}$ ft-poundals	1 ft-poundal	= 421,390 ergs
1 erg	$= 7.376 \times 10^{-8}$ ft-lb	1 ft-lb	= 1.35573 joules
1-g cm	$= 7.233 \times 10^{-5}$ ft-lb	1 ft-lb	= 13,825.5-g cm
1 joule	$= 10^7$ ergs		

9. POWER

1 watt	$= 10^7$ ergs/sec	1 ft.-poundal/sec	= 421,390 ergs/second
1 watt	= 23.731 ft.-poundals/sec	1 ft-lb/sec	= 1.35573 watts
1 watt	= 0.7376 ft.-lb/sec	1 hp	= 745.65 watts
1 watt	= 0.001341 hp	1 hp-sec	= 550 ft-lb
1 kw-hr	= 2,655,403 ft-lb	1 hp-sec	= 178.122 cal
1 kw-hr	= 1.3411 hp-hr		
1 kw-hr	= 859,975 cal		

10. HEAT

1 cal (g.c.)	= 0.0039683 B.t.u.	1 B.t.u.	= 252.00 cal
1 cal (g.c.)	= 4.1862 joules	1 B.t.u.	= 1055 joules
1 cal (g.c.)	= 3.088 ft-lb	1 B.t.u.	= 778.1 ft-lb
1 cal (g.c.)	= 0.005614 hp-sec	1 B.t.u.	= 1.4147 hp-sec

11. TIME

1 sidereal sec. = 0.99727 sec. (mean solar)
1 sec. (mean solar) = 1.002738 sidereal sec.
Length of seconds pendulum latitude 45° = 99.3555 cm = 39.1163 in.

12. LOGARITHM

$\log_{10} N$	$= 0.43429 \log_e N$	$\log_e N = 2.3026 \log_{10} N$
e	$= 2.7183$	

13. DATA WITH REGARD TO EARTH

Acceleration due to gravity (Minneapolis ϕ = 44° 58′.7, λ = 93° 13′.9, h = 256 meters)	= 980.596 cm/sec
Gravitation constant (c.g.s.units)	= 666.07×10^{-10}
Mean density of the earth	= 5.52470
Half diameter of earth (equatorial)	= 6378.2 km
Half diameter of earth (polar)	= 6356.5 km
Half diameter of earth (average)	= 6367.4 km
Half diameter of earth (average)	= 3956.5 miles
Mean distance from earth to sun	= 9.29×10^{7} miles
Mean distance from earth to moon	= 2.39×10^{5} miles
1° latitude at 40° latitude	= 69.0 miles

14. VELOCITY

Velocity of light per second	= 186,284 miles	= 299796 km
Velocity of sound per second (air 0°C.)	= 1089 ft	= 331.92 m
Velocity of sound per second (brass)	= 11,480 ft	= 3500 m
Velocity of sound per second (cast steel)	= 16,360 ft	= 4990 m
Velocity of sound per second (water)	= 4728 ft	= 1441 m

15. NUMERICAL VALUES OF THE FACTOR $\frac{0.6745}{\sqrt{n(n-1)}}$ *

	$\frac{0.6745}{\sqrt{n(n-1)}}$									
n	0	1	2	3	4	5	6	7	8	9
	......	......	0.4769	0.2754	0.1947	0.1508	0.1231	0.1041	0.0901	0.0795
10	0.0711	0.0643	0.0587	0.0540	0.0500	0.0465	0.0435	0.0409	0.0386	0.0365
20	0.0346	0.0329	0.0314	0.0300	0.0287	0.0275	0.0265	0.0255	0.0245	0.0237

* For use with Σv^2.

16. NUMERICAL VALUES OF THE FACTOR $\frac{0.8453}{n\sqrt{n-1}}$. *

	$\frac{0.8453}{n\sqrt{n-1}}$									
n	0	1	2	3	4	5	6	7	8	9
	......	......	0.4227	0.1993	0.1220	0.0845	0.0630	0.0493	0.0399	0.0332
10	00.282	0.0243	0.0212	0.0188	0.0167	0.0151	0.0136	0.0124	0.0114	0.0105
20	00.097	0.0090	0.0084	0.0078	0.0073	0.0069	0.0065	0.0061	0.0058	0.0055

* For use with Σd.

17. DATA WITH REGARD TO VARIOUS ELEMENTS

Element	Symbol	Valence	Relative atomic weight oxygen = 16	G/cc liquid or solid	Specific heat, 0°C.	Coeff. of linear expansion; mean 0 to 100°C. (40°C.*) × 10⁴	Specific resistance, microhms/cc at 0°C.	% increase of resistance for 1°C. at 20°C.	Electrochemical equivalent, g/coulomb × 1000	Melting point, °C.	Boiling point, °C.
			‡276	66	227	219	271	270	276	207	208
Aluminum	Al	3	27.1	2.58	0.209	0.222	2.8	......	0.0936	658	1800
Antimony	Sb	3	120.2	6.62	0.050	0.1056	40.6	0.389	0.4152	630	1440
Antimony	Sb	5	120.2	......	......	......	......	......	0.2491		
Argon	A	..	39.9	1.38	......	......	......	......	......	− 188	− 186 1
Barium	Ba	2	137.37	3.75	0.068	......	......	......	0.7118	850	
Bismuth	Bi	3	208.0	9.70	0.030	0.1316	108.7	0.354	0.7185	270	1430
Bismuth	Bi	5	208.0	......	......	......	......	......	0.4311		
Bromine	Br	1	79.92	3.15	0.084	......	......	......	0.8282	− 7.3	61.1
Cadmium	Cd	2	112.4	8.37	0.055	0.3159	......	......	0.5824	321	770
Calcium	Ca	2	40.09	1.52	0.157	......	......	......	0.2077	805	
Carbon	C	4	12.0	3.50	0.160	......	......	......	0.0313	3500	3600
Chlorine	Cl	1	35.46	1.51	0.226	......	......	......	0.3675	− 102	− 33.6
Chromium	Cr	3	52.0	6.92	0.104	......	......	......	0.1797	1505	2200
Chromium	Cr	6	52.0	......	......	......	......	......	0.090		
Cobalt	Co	2	58.97	8.71	0.103	0.1236*	9.8	......	0.3061	1490	
Cobalt	Co	3	58.97	......	......	......	......	......	0.2041		
Copper	Cu	1	63.57	8.87	0.0942	0.1666	1.64	0.388	0.6588	1083	2310

‡ Number of Smithsonian Table.

17. DATA WITH REGARD TO VARIOUS ELEMENTS.—(*Continued*)

Element	Symbol	Valence	Relative atomic weight oxygen = 16	G/cc liquid or solid	Specific heat, 0°C.	Coeff. of linear expansion; mean 0 to 100°C. (40°C.*) $\times 10^4$	Specific resistance, microhms/cc at 0°C.	% increase of resistance for 1°C. at 20°C.	Electrochemical equivalent, g/coulomb $\times 1000$	Melting point, °C.	Boiling point, °C.
Copper........	Cu	2	63.57	......	......	......	....	......	0.3290		
Fluorine......	F	1	19.0	1.14	......	......	....	......	0.1968	− 223	− 187
Gold..........	Au	3	197.2	19.3	0.0316	0.1470	2.07	0.365	0.6818	1063	
Helium........	He	...	4.0	......	......	......	....	......	......	....	− 267
Hydrogen.....	H	1	1.008	0.07	......	......	....	......	0.0104	− 259	− 252.6
Iodine.........	I	1	126.92	4.8	0.054	......	....	......	1.3153	114	200
Iridium........	Ir	4	193.1	22.4	0.0323	......	....	......	0.5003	2250	
Iron...........	Fe	2	55.85	7.87	0.1152	0.1089	9.7 to 12	0.513	0.2894	1520	2450
Iron...........	Fe	3	55.85	......	......	......	....	......	0.1929		
Lead...........	Pb	2	207.1	11.0	0.030	0.2709	18.4 to 19.6	0.387	1.0731	327	1525
Lithium........	Li	1	7.0	0.534	0.795	......	....	......	0.0725	186	1400
Magnesium....	Mg	2	24.32	1.70	0.249	0.2694*	4.1 to 5	......	0.1260	651	1120
Manganese.....	Mn	2	54.93	7.40	0.121	......	....	......	0.2846	1225	1900
Manganese.....	Mn	4	54.93	......	......	......	....	......	0.1423		
Mercury.......	Hg	1	200.0	13.596	0.0335	......	....	0.072	2.0727	− 39	357
Mercury.......	Hg	2	200.0	......	......	......	....	......	1.0363		
Neon..........	Ne	...	20.0	......	......	......	....	......	......	− 252	

17. DATA WITH REGARD TO VARIOUS ELEMENTS.—(*Continued*)

Element	Symbol	Valence	Relative atomic weight oxygen = 16	G/cc liquid or solid	Specific heat, 0°C.	Coeff. of linear expansion; mean 0 to 100°C. (40°C.*) × 10^4	Specific resistance, microhms/cc at 0°C.	% increase of resistance for 1°C. at 20°C.	Electrochemical equivalent, g/coulomb × 1000	Melting point, °C.	Boiling point, °C.
Nickel	Ni	2	58.68	8.80	0.109	0.1279*	11.1	......	0.3040	1450	
Nickel	Ni	3	58.68	......	......	......	......	......	0.2027		
Nitrogen	N	3	14.01	0.81	......	......	......	......	0.0484	− 211	− 195
Nitrogen	N	5	14.01	......	......	......	......	......	0.0290		
Oxygen	O	2	16.0	1.14	......	......	......	......	0.0829	− 230	− 182.7
Palladium	Pd	2	106.7	11.4	0.059	0.1176*	12.1	......	0.5528	1545	
Palladium	Pd	5	106.7	......	......	......	......	......	0.2211		
Phosphorus	P	3	31.0	2.34	0.20	1.253*	......	......	0.1071	44.2	288
Phosphorus	P	5	31.0	......	......	......	......	......	0.0642		
Platinum	Pt	2	195.0	21.5	0.0323	0.0899*	9.0 to 15.5	0.354	1.0104	1755	
Platinum	Pt	4	195.0	......	......	......	......	......	0.5052		
Potassium	K	1	39.1	0.85	0.170	......	25.1	0.4052	......	62.5	712
Radium	Ra	2	226.4	......	......	......	......	......	......		
Selenium	Se	2	79.2	4.5	0.068	0.6604	......	......	0.4104	......	690
Silicon	Si	4	28.3	2.2	0.1833	0.0763*	......	......	0.0733	1420	
Silver	Ag	1	107.88	10.5	0.0559	0.1921*	1.5 to 1.7	0.377	1.1180	961	1955
Sodium	Na	1	23.0	0.95	0.253	......	......	......	0.2384	97	750
Strontium	Sr	2	87.62	2.55	......	......	......	......	0.4540		

17. DATA WITH REGARD TO VARIOUS ELEMENTS.—(*Continued*)

Element	Symbol	Valence	Relative atomic weight oxygen = 16	G/cc liquid or solid	Specific heat, 0°C.	Coeff. of linear expansion; mean 0 to 100°C. (40°C.*) $\times 10^4$	Specific resistance, microhms/cc at 0°C.	% increase of resistance for 1°C. at 20°C.	Electrochemical equivalent, g/coulomb $\times$ 1000	Melting point, °C.	Boiling point, °C.
Sulphur	S	2	32.07	2.0	0.181	1.180	....	......	0.1662	115	444.7
Tantalum	Ta	5	181.0	10.6	0.0333	....	....	......	0.3751	2800	
Tellurium	Te	2	127.5	6.25	0.048	0.3687	....	......	0.6606	451	1390
Thallium	Tl	1	204.0	11.8	0.0326	0.3021*	....	......	2.1141	301	1700
Thorium	Th	2	232.42	11.0	0.0276	....	....	......	1.2043	1700	
Tin	Sn	2	119.0	7.18	0.055	0.2296	9.53	0.365	0.6166	231.9	2270
Tungsten	W	6	184.0	18.8	0.0336	....	....	......	0.3178	2950	
Uranium	U	2	238.5	18.7	0.028	....	....	......	1.2358	2000	
Uranium	U	3	238.5	......	......	....	....	......	0.8238		
Vanadium	V	3	51.2	5.5	0.1153	....	....	......	0.1768	1750	
Zinc	Zn	2	65.37	7.19	0.0935	0.2976	6.00	0.365	0.3385	419	930

18. ALLOYS

Alloy	Density	Specific heat Cal/g	Coef. of thermal expansion $\times 10^4/°C$	Specific resistance, microms	Temp. coef., %/°C
Brass (70 Cu 30 Zn)	8.44	0.0883	0.1859	8.0	0.40
German silver	8.3	0.0946	0.1836	31.5	0.025
Platinum and Iridium	21.62	......	0.0884	29.90	0.087
Constantine	8.97	......	....	48.30	0.0014
Manganin	8.6	......	....	39.14	0.0017
Advance	.....	......	....	50.0	6×10^{-6}
I_aI_a (soft)	.....	......	....	47.1	0.0005

19. GASES

Gas	Specific gravity	G/l	Lb/cu ft	Specific heat at constant pressure		Exp. coeff. 76 cm Hg. Constant vol. $\times 10^2$	Exp. coeff. 76 cm Hg. Constant pressure $\times 10^2$
	71*	71	71	232		223	
Air	1.000	1.2928	0.08071	(0-100°)	0.2374	0.36650	0.3671
Ammonia	0.597	0.7621	0.04758	(23–100)	0.5202		
Carbon dioxide	1.5291	1.9652	0.12269	(15-100)	0.2025	0.36856	0.3710
Carbon monoxide	0.9672	1.2506	0.07807	(23–99)	0.2425	0.36667	0.3669
Chlorine	2.491	3.1666	0.19769	(13–202)	0.1241		
Coal gas from	0.320	0.414	0.02583				
Coal gas to	0.740	0.957	0.05973				
Hydrogen	0.0696	0.09004	0 005621	(21–100)	3.410	0.36504	0.3660
Nitrogen	0.9673	1.2542	0.07829	(0–200)	0.2438	0.36682	
Oxygen	1.0530	1.4292	0.08922	(13–207)	0.2175	0.36681	
Steam at 100°	0.4690	0.5810	0.0363	100°	0.421	....	0.4187

* Number of Smithsonian Table.

20. DENSITY AND VOLUME OF WATER AT DIFFERENT TEMPERATURES

Smithsonian Table No. 75

°C.	Density	Volume	°C.	Density	Volume	°C.	Density	Volume	°C.	Density	Volume
0	0.99987	1.00013	20	0.99823	1.00177	40	0.99224	1.00782	80	0.97183	1.02899
+1	993	007	21	802	198	41	186	821	85	0.96865	1.03237
2	997	003	22	780	221	42	147	861	90	534	590
3	999	001	23	756	244	43	107	901	95	192	959
4	1.0000	1.0000	24	732	268	44	066	943	100	0.95838	1.04343
5	0.99999	1.0001	25	0.99707	1.00294	45	0.99025	1.00985	110	0.9510	1.0515
6	997	003	26	681	320	46	0.98982	1.01028	120	0.9434	1.0601
7	993	007	27	654	347	47	940	072	130	0.9352	1.0693
8	988	012	28	626	375	48	896	116	140	0.9264	1.0794
9	981	019	29	597	405	49	852	162	150	0.9173	1.0902
10	0.99973	1.00027	30	0.99567	1.00435	50	0.98807	1.01207	160	0.9075	1.1019
11	963	037	31	537	466	51	762	254	170	0.8973	1.1145
12	952	048	32	505	497	52	715	301	180	0.8866	1.1279
13	940	060	33	473	530	53	669	349	190	0.8750	1.1429
14	927	073	34	440	563	54	621	398	200	0.8628	1.1590
15	0.99913	1.00087	35	0.99406	1.00598	55	0.98573	1.01448	210	0.850	1.177
16	897	103	36	371	633	60	324	705	220	0.837	1.195
17	880	120	37	336	669	65	059	979	230	0.823	1.215
18	862	138	38	299	706	70	0.97781	1.02270	240	0.809	1.236
19	843	157	39	262	743	75	489	576	250	0.794	1.259

21. BOILING POINT OF WATER UNDER DIFFERENT BAROMETRIC PRESSURES

Bar. pressure	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
72.	8.50	98.54	98.57	98.61	98.65	98.69	98.73	98.77	98.80	98.84
73.	98.88	98.92	98.96	98.99	99.03	99.07	99.11	99.14	99.18	99.22
74.	99.26	99.30	99.33	99.37	99.41	99.44	99.48	99.52	99.56	99.59
75.	99.63	99.67	99.71	99.74	99.78	99.82	99.85	99.89	99.93	99.96
76.	100.00	100.04	100.07	100.11	100.15	100.18	100.22	100.26	100.29	100.33
77.	100.36	100.40	100.44	100.47	100.51	100.55	100.58	100.62	100.65	100.69

22. BAROMETER CORRECTION AT DIFFERENT TEMPERATURES, THE BAROMETER SCALE BEING CORRECT AT 62°F. (BRASS SCALE)

The correction is to be subtracted from the observed height

Temp., F°.	Pressure, inches			Temp., F°.	Pressure, inches		
	28	29	30		28	29	30
64	−0.090	0.093	0.096	74	−0.115	0.119	0.123
65	0.092	0.095	0.099	75	0.117	0.122	0.126
66	0.095	0.096	0.101	76	0.120	0.124	0.128
67	0.097	0.101	0.104	77	0.122	0.127	0.131
68	0.100	0.103	0.107	78	0.125	0.129	0.134
69	0.102	0.106	0.110	79	0.127	0.132	0.137
70	0.105	0.109	0.112	80	0.130	0.135	0.139
71	0.107	0.111	0.115	81	0.132	0.137	0.142
72	0.110	0.114	0.118	82	0.135	0.140	0.145
73	0.112	0.116	0.120	83	0.137	0.142	0.147

22a. BAROMETER CORRECTION. BRASS SCALE CORRECT AT 0°C., METRIC MEASURE

The correction is to be subtracted from the observed height

Height, mm	α mm for each °C.	Height, mm	α, mm for each °C.	Height, mm	α, mm for each °C.
500	0.0813	600	0.0975	700	0.1137
510	0.0830	610	0.0992	710	0.1154
520	0.0846	620	0.1008	720	0.1170
530	0.0862	630	0.1024	730	0.1186
540	0.0878	640	0.1040	740	0.1202
550	0.0894	650	0.1056	750	0.1218
560	0.0911	660	0.1073	760	0.1235
570	0.0927	670	0.1089	770	0.1251
580	0.0943	680	0.1105	780	0.1267
590	0.0959	690	0.1121	790	0.1283
				800	0.1290

23. PRESSURE OF SATURATED MERCURY VAPOR IN MILLIMETERS OF MERCURY

t°C.	Pressure	t°C.	Pressure	t°C.	Pressure	t°C.	Pressure	t°C.	Pressure
0	0.00047	10	0.00080	20	0.00133	70	0.050	120	0.779
2	0.00052	12	0.00089	30	0.0029	80	0.093	130	1.240
4	0.00058	14	0.00099	40	0.0063	90	0.165	140	1.930
6	0.00064	16	0.00109	50	0.0130	100	0.285	150	2.930
8	0.00072	18	0.00121	60	0.0260	110	0.478	160	4.380

24. SATURATED WATER VAPOR

Adapted from Smithsonian Tables

Showing pressure P (in millimeters of mercury) and density D in grams per cubic centimeter of aqueous vapor saturated at temperature t; or showing boiling point t of water and density D of steam corresponding to an outside pressure P.

t	P	D	t	P	D
−10	2.0	2.2×10^{-6}	80.0	355.1	293.8
− 9	2.1	2.4	85.0	433.5	354.1
− 8	2.3	2.6	90.0	525.8	424.1
− 7	2.6	2.8	91.0	546.1	439.5
− 6	2.8	3.0	92.0	567.1	455.2
− 5	3.0	3.3	93.0	588.7	471.3
− 4	3.3	3.5	94.0	611.0	487.8
− 3	3.6	3.8	95.0	634.0	505
− 2	3.9	4.1	96.0	657.7	523
− 1	4.2	4.5	96.5	669.8	
0	4.6	4.9	97.0	682.1	541
1	4.9	5.2	97.5	694.5	
2	5.3	5.6	98.0	707.3	560
3	5.7	5.9	98.2	712.5	
4	6.1	6.4	98.4	717.6	
5	6.5	6.8	98.6	722.8	
6	7.0	7.3	98.8	728.0	
7	7.5	7.8	99.0	733.3	579
8	8.0	8.3	99.2	738.6	
9	8.6	8.8	99.4	743.9	
10	9.2	9.4	99.6	749.3	
11	9.8	10.0	99.8	754.7	
12	10.5	10.7	100.0	760.0	598
13	11.2	11.4	100.2	765.5	
14	12.0	12.1	100.4	770.9	
15	12.8	12.8	100.6	776.4	
16	13.6	13.6	100.8	781.9	
17	14.5	14.5	101	787.5	618
18	15.5	15.4	102	815.9	639
19	16.5	16.3	103	845.1	661
20	17.6	17.3	104	875.1	683
21	18.7	18.3	105	906.1	705
22	19.8	19.4	106	937.9	728
23	21.1	20.6	107	970.6	751
24	22.4	21.8	108	1004.3	776
25	23.8	23.0	109	1038.8	801
26	25.2	24.4	110	1074.5	827
27	26.8	25.8	112	1148.7	880
28	28.4	27.2	114	1227.1	936
29	30.1	28.8	116	1309.8	995
30	31.8	30.4	118	1397.0	1057
35	42.0	39.6	120	1489	1122
40	55.1	51.1	125	1740	1299
45	71.7	65.6	130	2026	1498
50	92.3	83.2	135	2348	1721
55	117.8	104.6	140	2710	1968
60	149.2	130.5	150	3569	2550
65	187.4	161.5	160	4633	3265
70	233.5	198.4	175	6689	4621
75	289.0	242.1	200	11650	7840

25. THE WET- AND DRY-BULB HYGROMETER

Adapted from Smithsonian Tables

Let the temperature of the atmosphere given by a dry-bulb thermometer be denoted by t°C., and let the reading of the wet-bulb thermometer be denoted by $(t - \Delta t)$. In the following table, corresponding to the various values of Δt given in the top line, the pressure (in millimeters of mercury) of the aqueous vapor in the atmosphere at the temperature t°C. is given.

	Difference between the dry- and wet-bulb readings											
t°C.	0	1	2	3	4	5	6	7	8	9	10	12	14
−5	3.0	2.3	1.6	0.8	0.2								
−4	3.3	2.5	1.8	1.0	0.4								
−3	3.6	2.8	2.0	1.2	0.7								
−2	3.9	3.1	2.3	1.6	0.8								
−1	4.2	3.4	2.6	1.8	1.0								
0	4.6	3.7	2.9	2.1	1.3								
1	4.9	4.1	3.2	2.4	1.6								
2	5.3	4.4	3.6	2.7	1.9	1.1	0.3						
3	5.7	4.8	3.9	3.1	2.2	1.4	0.6						
4	6.1	5.2	4.3	3.4	2.6	1.8	0.9						
5	6.5	5.6	4.7	3.8	2.9	2.1	1.2						
6	7.0	6.0	5.1	4.2	3.3	2.4	1.6						
7	7.5	6.5	5.5	4.6	3.7	2.8	1.9	1.1	0.2				
8	8.0	7.0	6.0	5.0	4.1	3.2	2.3	1.4	0.6				
9	8.6	7.5	6.5	5.5	4.5	3.6	2.7	1.8	0.9				
10	9.2	8.1	7.0	6.0	5.0	4.0	3.1	2.2	1.3				
11	9.8	8.7	7.6	6.5	5.5	4.5	3.5	2.6	1.7				
12	10.5	9.3	8.2	7.1	6.0	5.0	4.0	3.0	2.1	1.2	0.3		
13	11.2	10.0	8.8	7.6	6.6	5.5	4.5	3.5	2.5	1.6	0.6		
14	12.0	10.8	9.5	8.4	7.2	6.2	5.0	4.0	3.0	2.0	1.1		
15	12.8	11.5	10.2	9.1	7.9	6.7	5.5	4.5	3.5	2.5	1.5		
16	13.6	12.3	11.0	9.8	8.5	7.3	6.2	5.1	4.0	3.0	2.0	0.1	
17	14.5	13.1	11.8	10.5	9.2	8.1	6.8	5.7	4.6	3.6	2.5	0.5	
18	15.5	14.0	12.6	11.3	10.0	8.7	7.5	6.4	5.2	4.1	3.0	1.0	
19	16.5	15.0	13.5	12.1	10.8	9.4	8.2	6.9	5.8	4.6	3.5	1.4	
20	17.6	16.1	14.5	13.0	11.6	10.3	8.9	7.6	6.4	5.2	4.1	2.0	
21	18.7	17.1	15.5	13.9	12.5	11.1	9.7	8.5	7.2	6.0	4.8	2.5	0.4
22	19.8	18.1	16.5	14.9	13.4	12.0	10.6	9.2	7.9	6.6	5.4	3.1	1.0
23	21.1	19.3	17.6	16.0	14.4	12.9	11.5	10.1	8.7	7.4	6.1	3.8	1.5
24	22.4	20.6	18.8	17.2	15.5	14.0	12.4	11.0	9.5	8.2	6.9	4.4	2.1
25	23.8	21.9	20.1	18.3	16.6	15.0	13.4	11.9	10.4	9.1	7.7	5.1	2.7
26	25.2	23.3	21.4	19.6	17.8	16.1	14.5	13.0	11.4	9.9	8.5	5.9	3.4
27	26.8	24.8	22.8	21.0	19.0	17.3	15.6	14.0	12.4	10.9	9.4	6.7	4.1
28	28.4	26.3	24.2	22.2	20.3	18.5	16.8	15.1	13.4	11.9	10.4	7.5	4.9
29	30.1	27.9	25.7	23.7	21.7	19.8	18.0	16.3	14.6	13.0	11.4	8.4	5.6
30	31.9	29.6	27.3	25.3	23.2	21.2	19.3	17.5	15.7	14.0	12.4	9.3	6.5

26. WAVE LENGTHS IN ÅNGSTRÖM UNITS (10^{-8} cm)

Smithsonian Table No. 151

Barium	5535.7	Hydrogen	4101.8	Potassium	7668.5
Caesium	4555.4	Hydrogen	4340.7	Potassium	7701.9
Caesium	4593.3	Hydrogen	4861.5	Rubidium	6298.7
Calcium	5589.0	Hydrogen	6563.0	Sodium	5890.2
Cadmium	4799.9	Lithium	6708.2	Sodium	5896.2
Cadmium	5085.8	Magnesium	5167.5	Strontium	4607.5
Cadmium	6438.5	Magnesium	5172.9	Strontium	5481.2
Helium	5875.8	Magnesium	5183.8	Strontium	6408.6
Helium	5876.2	Mercury	5460.7	Thallium	5350.6

27. INDEX OF REFRACTION

Material	*D* line		*D* line
Water	1.3335	Benzol	1.5014
Alcohol	1.3624	Ether, ethyl	1.3560
Carbon bisulphide	1.6293	Chloroform	1.4490
Glass, light crown (δ = 2.50)	1.5280	Glycerin	1.4729
Glass, heavy crown (δ = 3.00)	1.5604	Aluminum	1.44
Glass, light flint (δ = 2.87)	1.5410	Copper	0.641
Glass, heavy flint (δ = 4.22)	1.7102	Gold	0.366
Quartz, crystal + ord	1.5442	Iron	2.36
Quartz, crystal + ext	1.5533	Silver	0.181
Iceland spar, ord	1.6585	Antimony	3.04
Iceland spar, ext	1.4864	Air, 0°76 cm	1.000293

28. ELASTIC CONSTANTS

Dynes per square centimeter

Material	Young's modulus $\times 10^{-11}$	Rigidity $\times 10^{-11}$	Elastic limit $\times 10^{-8}$	Velocity of sound cm/sec $\times 10^{-5}$
Aluminum	6.5	2.4–3.3	....	5.10
Brass	7.7– 9.3	3.1–3.6	4.5–11	3.56
Copper	8.5–11.6	3.2–4.5	3– 7	3.74
Iron, cast	6.9–10.8	2.6–4.2	7	4.32
Iron, wrought	19.3–20.9	7.7–8.5	20	5.06
Iron, steel	20.1–21.6	8.0–8.8	33–40	5.22
Platinum	15	6.1–6.5	14–26	2.69
Silver	7	2.6	3–11	2.61
Wood	0.7– 1.5	0.07–0.12	1.5–2.4	3.0

29. COMPRESSIBILITY

Material	Compression per megabarye	Elasticity of volume
Water	5.0×10^{-5}	2.03×10^{10}
Mercury	3.9×10^{-6}	2.60×10^{11}
Glass	2.6×10^{-6}	3.90×10^{11}

30. FUNDAMENTAL ELECTRICAL UNITS

Magnetic Units

Magnetic pole of unit strength is a point pole which in a vacuum acts on another equal point pole one centimeter distant with a force of one dyne.

Magnetic field of unit intensity, the oersted, is a field that acts on a unit magnetic point pole placed in it with a force of one dyne.

Electrostatic Units

Electrostatic unit of quantity of electricity, *the statcoulomb,* is that quantity which, when concentrated at a point in a vacuum, acts on an equal point charge one centimeter distant with a force of one dyne.

Electrostatic unit of current, the statampere, exists in a circuit when one statcoulomb per second is passing through every plane of the circuit.

Electrostatic unit of potential, the statvolt, is the potential at any point when it requires one erg of work to transfer one statcoulomb to the point from an infinite distance or from a point of zero potential.

Potential difference of one statvolt exists between two points when it requires one erg of work to carry one statcoulomb from one of the points to the other.

Electromagnetic Units

Electromagnetic unit of current, the abampere, is that current which in a circular conductor of one-centimeter radius produces a magnetic field of one oersted at the center for each centimeter length of the conductor.

Electromagnetic unit of quantity, the abcoulomb, is that quantity of electricity which passes any given cross section per second

when the current in the circuit is one abampere. (This unit has been found by experiment to be 3×10^{10} times larger than the statcoulomb.)

Electromagnetic unit of potential difference, the abvolt, exists between two points when one erg of work is expended in carrying one abcoulomb from one of the points to the other. (This unit therefore is smaller than the statvolt by the factor 3×10^{10}.)

Electromagnetic unit of resistance, the abohm, is the resistance in which a potential difference of one abvolt produces a current of one abampere.

Practical Units

Ampere is one-tenth of an abampere.

Coulomb is that quantity of electricity which passes any given plane in one second when the current is one ampere.

Volt is equal to 10^8 abvolts, or is the potential difference through which the transfer of one coulomb requires the expenditure of one joule of energy. (Three hundred volts therefore equal one statvolt.)

Ohm is the resistance in which a potential difference of one volt produces a current of one ampere. (One ohm therefore equals 10^9 abohms.)

Joule is equal to 10^7 ergs of work or of energy. It is also equal to the energy expended in moving one coulomb through a potential difference of one volt. $W_J = EIt$ joules.

Watt is an electric unit of power and represents work done at the rate of one joule per second. $P = EI$ watts. A kilowatt = 1000 watts.

Farad is the practical unit of capacitance. A body or a condenser has a capacitance of one farad when a transfer of one coulomb produces a change of one volt in its potential or potential difference. *One microfarad* $\equiv$ 0.000001 farad.

Henry is the practical unit of inductance. The inductance is one henry if an e.m.f. of one volt is induced when the inducing current changes at the rate of one ampere per second. This applies to both self- and mutual inductance. A *millihenry* $\equiv$ 0.001 henry.

Oersted is the practical unit of magnetic-field intensity. The intensity is one oersted when the field acts on a unit point pole

placed in it with a force of one dyne (as already stated under Magnetic Units).

Maxwell is the practical unit of magnetic flux. It represents a single magnetic line of force. The flux in a magnetic field is Φ maxwells when it contains Φ lines of force. $\Phi = A\mathcal{H}$, where A is the area of the field in square centimeters, and $\mathcal{H}$ the average intensity of the field in oersteds. By convention, a magnetic field whose intensity is one oersted is assigned one magnetic line (maxwell) per square centimeter.

Gauss is the unit of density of magnetic flux and in open space is numerically equal to the field intensity in oersteds.

31. DEFINITIONS OF THE FUNDAMENTAL ELECTRICAL UNITS BY THE LONDON CONFERENCE (1908)

These fundamental units are:

1. The *ohm*, the unit of electric resistance, which has the value of 1,000,000,000 (10^9) in terms of the centimeter and the second.

2. The *ampere*, the unit of electric current, which has the value of one-tenth (0.1) in terms of the centimeter, gram, and second.

3. The *volt*, the unit of e.m.f., which has the value of 100,000,000 (10^8) in terms of the centimeter, gram, and second.

4. The *watt*, the unit of power, which has the value 10,000,000 (10^7) in terms of the centimeter, the gram, and the second.

As a system of units representing the foregoing and sufficiently near for the purpose of electrical measurements, and as a basis for legislation, the conference recommended the adoption of the international ohm, the international ampere, the international volt, and the international watt, defined as follows:

1. The *international ohm* is the resistance offered to an unvarying electric current by a column of mercury at the temperature of melting ice (14.4521-g in mass) of a constant cross-sectional area and 106.300 cm in length.

2. The *international ampere* is the unvarying electric current which, when passed through a solution of nitrate of silver in water, deposits silver at the rate of 0.00111800-g/second.

3. The *international volt* is the electrical pressure which, when steadily applied to a conductor whose resistance is one international ohm, produces a current of one international ampere.

4. The *international watt* is the energy expended per second by an unvarying electric current of one international ampere under an electric pressure of one international volt.

32. THE ELECTROMOTIVE FORCE OF THE WESTON NORMAL CELL

The e.m.f. of the Weston normal cell derived from the international ohm and the international ampere according to the resolutions of the London Conference is, within 1 part in 10,000,

$$E = 1.01830 \text{ international volts at } 20°\text{C}.$$

The formula for the temperature coefficient of the Weston normal cell adopted by the London Conference, based on the investigations of the U.S. Bureau of Standards, is as follows:

$$E_t = E_{20} - 0.0000406\,(t - 20°) - 0.00000095\,(t - 20)^2 + 0.00000001\,(t - 20)^3.$$

33. RELATION BETWEEN ELECTRICAL UNITS

1 amp. $\equiv$ 0.1 electromagnetic units $\equiv 3 \times 10^9$ electrostatic units.

1 volt $\equiv 10^8$ electromagnetic units $\equiv \frac{1}{300}$ electrostatic units.

1 ohm $\equiv 10^9$ electromagnetic units $\equiv \frac{1}{9 \times 10^{11}}$ electrostatic units.

1 farad $\equiv \frac{1}{10^9}$ electromagnetic units $\equiv 9 \times 10^{11}$ electrostatic units.

1 henry $\equiv 10^9$ electromagnetic units $\equiv \frac{1}{9 \times 10^{11}}$ electrostatic units.

TABLES

Table I.—Logarithms*

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7

* From "Experiments in Physics," Ingersoll and Martin.

TABLE I.—LOGARITHMS—*Continued*

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

TABLE II.—RECIPROCALS*

	0	1	2	3	4	5	6	7	8	9
.0	....	100.0	50.00	33.33	25.00	20.00	16.67	14.29	12.50	11.11
.1	10.0000	9.091	8.333	7.692	7.143	6.667	6.250	5.882	5.556	5.263
.2	5.0000	4.762	4.545	4.348	4.167	4.000	3.846	3.704	3.571	3.448
.3	3.3333	3.226	3.125	3.030	2.941	2.857	2.778	2.703	2.632	2.564
.4	2.5000	2.439	2.381	2.326	2.273	2.222	2.174	2.128	2.083	2.041
.5	2.0000	*9608	*9231	*8868	*8519	*8182	*7857	*7544	*7241	*6949
.6	1.6667	6393	6129	5873	5625	5385	5152	4925	4706	4493
.7	4286	4085	3889	3699	3514	3333	3158	2987	2821	2658
.8	2500	2346	2195	2048	1905	1765	1628	1494	1364	1236
.9	1111	0989	0870	0753	0638	0526	0417	0309	0204	0101
1.0	1.0000	*9901	*9804	*9709	*9615	*9524	*9434	*9346	*9259	*9174
1.1	0.9091	9009	8929	8850	8772	8696	8621	8547	8475	8403
1.2	8333	8264	8197	8130	8065	8000	7937	7874	7813	7752
1.3	7692	7634	7576	7519	7463	7407	7353	7299	7246	7194
1.4	7143	7092	7042	6993	6944	6897	6849	6803	6757	6711
1.5	6667	6623	6579	6536	6494	6452	6410	6369	6329	6289
1.6	6250	6211	6173	6135	6098	6061	6024	5988	5952	5917
1.7	5882	5848	5814	5780	5747	5714	5682	5650	5618	5587
1.8	5556	5525	5495	5464	5435	5405	5376	5348	5319	5291
1.9	5263	5236	5208	5181	5155	5128	5102	5076	5051	5025
2.0	0.5000	4975	4950	4926	4902	4878	4854	4831	4808	4785
2.1	4762	4739	4717	4695	4673	4651	4630	4608	4587	4566
2.2	4545	4525	4505	4484	4464	4444	4425	4405	4386	4367
2.3	4348	4329	4310	4292	4274	4255	4237	4219	4202	4184
2.4	4167	4149	4132	4115	4098	4082	4065	4049	4032	4016
2.5	4000	3984	3968	3953	3937	3922	3906	3891	3876	3861
2.6	3846	3831	3817	3802	3788	3774	3759	3745	3731	3717
2.7	3704	3690	3676	3663	3650	3636	3623	3610	3597	3584
2.8	3571	3559	3546	3534	3521	3509	3497	3484	3472	3460
2.9	3448	3436	3425	3413	3401	3390	3378	3367	3356	3344
3.0	0.3333	3322	3311	3300	3289	3279	3268	3257	3247	3236
3.1	3226	3215	3205	3195	3185	3175	3165	3155	3145	3135
3.2	3125	3115	3106	3096	3086	3077	3067	3058	3049	3040
3.3	3030	3021	3012	3003	2994	2985	2976	2967	2959	2950
3.4	2941	2933	2924	2915	2907	2899	2890	2882	2874	2865
3.5	2857	2849	2841	2833	2825	2817	2809	2801	2793	2786
3.6	2778	2770	2762	2755	2747	2740	2732	2725	2717	2710
3.7	2703	2695	2688	2681	2674	2667	2660	2653	2646	2639
3.8	2632	2625	2618	2611	2604	2597	2591	2584	2577	2571
3.9	2564	2558	2551	2545	2538	2532	2525	2519	2513	2506
4.0	0.2500	2494	2488	2481	2475	2469	2463	2457	2451	2445
4.1	2439	2433	2427	2421	2415	2410	2404	2398	2392	2387
4.2	2381	2375	2370	2364	2358	2353	2347	2342	2336	2331
4.3	2326	2320	2315	2309	2304	2299	2294	2288	2283	2278
4.4	2273	2268	2262	2257	2252	2247	2242	2237	2232	2227
4.5	2222	2217	2212	2208	2203	2198	2193	2188	2183	2179
4.6	2174	2169	2165	2160	2155	2151	2146	2141	2137	2132
4.7	2128	2123	2119	2114	2110	2105	2101	2096	2092	2088
4.8	2083	2079	2075	2070	2066	2062	2058	2053	2049	2045
4.9	2041	2037	2033	2028	2024	2020	2016	2012	2008	2004

Differences

	98	96	94	92	90
1	10	10	9	9	9
2	20	19	19	18	18
3	29	29	28	28	27
4	39	38	38	37	36
5	49	48	47	46	45
6	59	58	56	55	54
7	69	67	66	64	63
8	78	77	75	74	72
9	88	86	85	83	81
	88	86	84	82	80
1	9	9	8	8	8
2	18	17	17	16	16
3	26	26	25	25	24
4	35	34	34	33	32
5	44	43	42	41	40
6	53	52	50	49	48
7	62	60	59	57	56
8	70	69	67	66	64
9	79	77	76	74	72
	78	76	74	72	70
1	8	8	7	7	7
2	16	15	15	14	14
3	23	23	22	22	21
4	31	30	30	29	28
5	39	38	37	36	35
6	47	46	44	43	42
7	55	53	52	50	49
8	62	61	59	58	56
9	70	68	67	65	63
	68	66	64	62	60
1	7	7	6	6	6
2	14	13	13	12	12
3	20	20	19	19	18
4	27	26	26	25	24
5	34	33	32	31	30
6	41	40	38	37	36
7	48	46	45	43	42
8	54	53	51	50	48
9	61	59	58	56	54
	58	56	54	52	50
1	6	6	5	5	5
2	12	11	11	10	10
3	17	17	16	16	15
4	23	22	22	21	20
5	29	28	27	26	25
6	35	34	32	31	30
7	41	39	38	36	35
8	46	45	43	42	40
9	52	50	49	47	45
	48	46	44	43	42
1	5	5	4	4	4
2	10	9	9	9	8
3	14	14	13	13	13
4	19	18	18	17	17
5	24	23	22	22	21
6	29	28	26	26	25
7	34	32	31	30	29
8	38	37	35	34	34
9	43	41	40	39	38
	41	40	39	38	37
1	4	4	4	4	4
2	8	8	8	8	7
3	12	12	12	11	11
4	16	16	16	15	15
5	21	20	20	19	19
6	25	24	23	23	22
7	29	28	27	27	26
8	33	32	31	30	30
9	37	36	35	34	33

* From "Experiments in Physics," Ingersoll and Martin.

TABLE II.—RECIPROCALS—*Continued*

	0	1	2	3	4	5	6	7	8	9
5.0	0.2000	1996	1992	1988	1984	1980	1976	1972	1969	1965
5.1	1961	1957	1953	1949	1946	1942	1938	1934	1931	1927
5.2	1923	1919	1916	1912	1908	1905	1901	1898	1894	1890
5.3	1887	1883	1880	1876	1873	1869	1866	1862	1859	1855
5.4	1852	1848	1845	1842	1838	1835	1832	1828	1825	1821
5.5	1818	1815	1812	1808	1805	1802	1799	1795	1792	1789
5.6	1786	1783	1779	1776	1773	1770	1767	1764	1761	1757
5.7	1754	1751	1748	1745	1742	1739	1736	1733	1730	1727
5.8	1724	1721	1718	1715	1712	1709	1706	1704	1701	1698
5.9	1695	1692	1689	1686	1684	1681	1678	1675	1672	1669
6.0	0.1667	1664	1661	1658	1656	1653	1650	1647	1645	1642
6.1	1639	1637	1634	1631	1629	1626	1623	1621	1618	1616
6.2	1613	1610	1608	1605	1603	1600	1597	1595	1592	1590
6.3	1587	1585	1582	1580	1577	1575	1572	1570	1567	1565
6.4	1563	1560	1558	1555	1553	1550	1548	1546	1543	1541
6.5	1538	1536	1534	1531	1529	1527	1524	1522	1520	1517
6.6	1515	1513	1511	1508	1506	1504	1502	1499	1497	1495
6.7	1493	1490	1488	1486	1484	1481	1479	1477	1475	1473
6.8	1471	1468	1466	1464	1462	1460	1458	1456	1453	1451
6.9	1449	1447	1445	1443	1441	1439	1437	1435	1433	1431
7.0	0.1429	1427	1425	1422	1420	1418	1416	1414	1412	1410
7.1	1408	1406	1404	1403	1401	1399	1397	1395	1393	1391
7.2	1389	1387	1385	1383	1381	1379	1377	1376	1374	1372
7.3	1370	1368	1366	1364	1362	1361	1359	1357	1355	1353
7.4	1351	1350	1348	1346	1344	1342	1340	1339	1337	1335
7.5	1333	1332	1330	1328	1326	1325	1323	1321	1319	1318
7.6	1316	1314	1312	1311	1309	1307	1305	1304	1302	1300
7.7	1299	1297	1295	1294	1292	1290	1289	1287	1285	1284
7.8	1282	1280	1279	1277	1276	1274	1272	1271	1269	1267
7.9	1266	1264	1263	1261	1259	1258	1256	1255	1253	1252
8.0	0.1250	1248	1247	1245	1244	1242	1241	1239	1238	1236
8.1	1235	1233	1232	1230	1229	1227	1225	1224	1222	1221
8.2	1220	1218	1217	1215	1214	1212	1211	1209	1208	1206
8.3	1205	1203	1202	1200	1199	1198	1196	1195	1193	1192
8.4	1190	1189	1188	1186	1185	1183	1182	1181	1179	1178
8.5	1176	1175	1174	1172	1171	1170	1168	1167	1166	1164
8.6	1163	1161	1160	1159	1157	1156	1155	1153	1152	1151
8.7	1149	1148	1147	1145	1144	1143	1142	1140	1139	1138
8.8	1136	1135	1134	1133	1131	1130	1129	1127	1126	1125
8.9	1124	1122	1121	1120	1119	1117	1116	1115	1114	1112
9.0	0.1111	1110	1109	1107	1106	1105	1104	1103	1101	1100
9.1	1099	1098	1096	1095	1094	1093	1092	1091	1089	1088
9.2	1087	1086	1085	1083	1082	1081	1080	1079	1078	1076
9.3	1075	1074	1073	1072	1071	1070	1068	1067	1066	1065
9.4	1064	1063	1062	1060	1059	1058	1057	1056	1055	1054
9.5	1053	1052	1050	1049	1048	1047	1046	1045	1044	1043
9.6	1042	1041	1040	1038	1037	1036	1035	1034	1033	1032
9.7	1031	1030	1029	1028	1027	1026	1025	1024	1022	1021
9.8	1020	1019	1018	1017	1016	1015	1014	1013	1012	1011
9.9	1010	1009	1008	1007	1006	1005	1004	1003	1002	1001

Differences

	36	35	34	33	32
1	4	4	3	3	3
2	7	7	7	7	6
3	11	11	10	10	10
4	14	14	14	13	13
5	18	18	17	17	16
6	22	21	20	20	19
7	25	25	24	23	22
8	29	28	27	26	26
9	32	32	31	30	29
	31	30	29	28	27
1	3	3	3	3	3
2	6	6	6	6	5
3	9	9	9	8	8
4	12	12	12	11	11
5	16	15	15	14	14
6	19	18	17	17	16
7	22	21	20	20	19
8	25	24	23	22	22
9	28	27	26	25	24
	26	25	24	23	22
1	3	3	2	2	2
2	5	5	5	5	4
3	8	8	7	7	7
4	10	10	10	9	9
5	13	13	12	12	11
6	16	15	14	14	13
7	18	18	17	16	15
8	21	20	19	18	18
9	23	23	22	21	20
	21	20	19	18	17
1	2	2	2	2	2
2	4	4	4	4	3
3	6	6	6	5	5
4	8	8	8	7	7
5	11	10	10	9	9
6	13	12	11	11	10
7	15	14	13	13	12
8	17	16	15	14	14
9	19	18	17	16	15
	10	15	14	13	12
1	2	2	1	1	1
2	3	3	3	3	2
3	5	5	4	4	4
4	6	6	6	5	5
5	8	8	7	7	6
6	10	9	8	8	7
7	11	11	10	9	8
8	13	12	11	10	10
9	14	14	13	12	11
	11	10	9	8	7
1	1	1	1	1	1
2	2	2	2	2	1
3	3	3	3	2	2
4	4	4	4	3	3
5	6	5	5	4	4
6	7	6	5	5	4
7	8	7	6	6	5
8	9	8	7	6	6
9	10	9	8	7	6
	6	5	4	3	2
1	1	1	0	0	0
2	1	1	1	1	0
3	2	2	1	1	1
4	2	2	2	1	1
5	3	3	2	2	1
6	4	3	2	2	1
7	4	4	3	2	1
8	5	4	3	2	2
9	5	5	4	3	2

TABLE III.—SQUARES*

0	0	1	2	3	4	5	6	7	8	9
0	0	1	4	9	16	25	36	49	64	81
1	100	121	144	169	196	225	256	289	324	361
2	400	441	484	529	576	625	676	729	784	841
3	900	961	1024	1089	1156	1225	1296	1369	1444	1521
4	1600	1681	1764	1849	1936	2025	2116	2209	2304	2401
5	2500	2601	2704	2809	2916	3025	3136	3249	3364	3481
6	3600	3721	3844	3969	4096	4225	4356	4489	4624	4761
7	4900	5041	5184	5329	5476	5625	5776	5929	6084	6241
8	6400	6561	6724	6889	7056	7225	7396	7569	7744	7921
9	8100	8281	8464	8649	8836	9025	9216	9409	9604	9801
10	1 0000	1 0201	1 0404	1 0609	1 0816	1 1025	1 1236	1 1449	1 1664	1 1881
11	2100	2321	2544	2769	2996	3225	3456	3689	3924	4161
12	4400	4641	4884	5129	5376	5625	5876	6129	6384	6641
13	6900	7161	7424	7689	7956	8225	8496	8769	9044	9321
14	9600	9881	2 0164	2 0449	2 0736	2 1025	2 1316	2 1609	2 1904	2 2201
15	2 2500	2 2801	3104	3409	3716	4025	4336	4649	4964	5281
16	5600	5921	6244	6569	6896	7225	7556	7889	8224	8561
17	8900	9241	9584	9929	3 0276	3 0625	3 0976	3 1329	3 1684	3 2041
18	3 2400	3 2761	3 3124	3 3489	3856	4225	4596	4969	5344	5721
19	6100	6481	6864	7249	7636	8025	8416	8809	9204	9601
20	4 0000	4 0401	4 0804	4 1209	4 1616	4 2025	4 2436	4 2849	4 3264	4 3681
21	4100	4521	4944	5369	5796	6225	6656	7089	7524	7961
22	8400	8841	9284	9729	5 0176	5 0625	5 1076	5 1529	5 1984	5 2441
23	5 2900	5 3361	5 3824	5 4289	4756	5225	5696	6169	6644	7121
24	7600	8081	8564	9049	9536	6 0025	6 0516	6 1009	6 1504	6 2001
25	6 2500	6 3001	6 3504	6 4009	6 4516	5025	5536	6049	6564	7081
26	7600	8121	8644	9169	9696	7 0225	7 0756	7 1289	7 1824	7 2361
27	7 2900	7 3441	7 3984	7 4529	7 5076	5625	6176	6729	7284	7841
28	8400	8961	9524	8 0089	8 0656	8 1225	8 1796	8 2369	8 2944	8 3521
29	8 4100	8 4681	8 5264	5849	6436	7025	7616	8209	8804	9401
30	9 0000	9 0601	9 1204	9 1809	9 2416	9 3025	9 3636	9 4249	9 4864	9 5481
31	6100	6721	7344	7969	8596	9225	9856	10 0489	10 1124	10 1761
32	10 2400	10 3041	10 3684	10 4329	10 4976	10 5625	10 6276	6929	7584	8241
33	8900	9561	11 0224	11 0889	11 1556	11 2225	11 2896	11 3569	11 4244	11 4921
34	11 5600	11 6281	6964	7649	8336	9025	9716	12 0409	12 1104	12 1801
35	12 2500	12 3201	12 3904	12 4609	12 5316	12 6025	12 6736	7449	8164	8881
36	9600	13 0321	13 1044	13 1769	13 2496	13 2225	13 3956	13 4689	13 5424	13 6161
37	13 6900	7641	8384	9129	9876	14 0625	14 1376	14 2129	14 2884	14 3641
38	14 4400	14 5161	14 5924	14 6689	14 7456	8225	8996	9769	15 0544	15 1321
39	15 2100	15 2881	15 3664	15 4449	15 5236	15 6025	15 6816	15 7609	8404	9201
40	16 0000	16 0801	16 1604	16 2409	16 3216	16 4025	16 4836	16 5649	16 6464	16 7281
41	8100	8921	9744	17 0569	17 1396	17 2225	17 3056	17 3889	17 4724	17 5561
42	17 6400	17 7241	17 8084	8929	9776	18 0625	18 1476	18 2329	18 3184	18 4041
43	18 4900	18 5761	18 6624	18 7489	18 8356	9225	19 0096	19 0969	19 1844	19 2721
44	19 3600	19 4481	19 5364	19 6249	19 7136	19 8025	8916	9809	20 0704	20 1601
45	20 2500	20 3401	20 4304	20 5209	20 6116	20 7025	20 7936	20 8849	9764	21 0681
46	21 1600	21 2521	21 3444	21 4369	21 5296	21 6225	21 7156	21 8089	21 9024	9961
47	22 0900	22 1841	22 2784	22 3729	22 4676	22 5625	22 6576	22 7529	22 8484	22 9441
48	23 0400	23 1361	23 2324	23 3289	23 4256	23 5225	23 6196	23 7169	23 8144	23 9121
49	24 0100	24 1081	24 2064	24 3049	24 4036	24 5025	24 6016	24 7009	24 8004	24 9001
50	0	1	2	3	4	5	6	7	8	9

* From "Experiments in Physics," Ingersoll and Martin.

Table III.—Squares—*Continued*

50	0	1	2	3	4	5	6	7	8	9
50	25 0000	25 1001	25 2004	25 3009	25 4016	25 5025	25 6036	25 7049	25 8064	25 9081
51	26 0100	26 1121	26 2144	26 3169	26 4196	26 5225	26 6256	26 7289	26 8324	26 9361
52	27 0400	27 1441	27 2484	27 3529	27 4576	27 5625	27 6676	27 7729	27 8784	27 9841
53	28 0900	28 1961	28 3024	28 4089	28 5156	28 6225	28 7296	28 8369	28 9444	29 0521
54	29 1600	29 2681	29 3764	29 4849	29 5936	29 7025	29 8116	29 9209	30 0304	30 1401
55	30 2500	30 3601	30 4704	30 5809	30 6916	30 8025	30 9136	31 0249	31 1364	31 2481
56	31 3600	31 4721	31 5844	31 6969	31 8096	31 9225	32 0356	32 1489	32 2624	32 3761
57	32 4900	32 6041	32 7184	32 8329	32 9476	33 0625	33 1776	33 2929	33 4084	33 5241
58	33 6400	33 7561	33 8724	33 9889	34 1056	34 2225	34 3396	34 4569	34 5744	34 6921
59	34 8100	34 9281	35 0464	35 1649	35 2836	35 4025	35 5216	35 6409	35 7604	35 8801
60	36 0000	36 1201	36 2404	36 3609	36 4816	36 6025	36 7236	36 8449	36 9664	37 0881
61	37 2100	37 3321	37 4544	37 5769	37 6996	37 8225	37 9456	38 0689	38 1924	38 3161
62	38 4400	38 5641	38 6884	38 8129	38 9376	39 0625	39 1876	39 3129	39 4384	39 5641
63	39 6900	39 8161	39 9424	40 0689	40 1956	40 3225	40 4496	40 5769	40 7044	40 8321
64	40 9600	41 0881	41 2164	41 3449	41 4736	41 6025	41 7316	41 8609	41 9904	42 1201
65	42 2500	42 3801	42 5104	42 6409	42 7716	42 9025	43 0336	43 1649	43 2964	43 4281
66	43 5600	43 6921	43 8244	43 9569	44 0896	44 2225	44 3556	44 4889	44 6224	44 7561
67	44 8900	45 0241	45 1584	45 2929	45 4276	45 5625	45 6976	45 8329	45 9684	46 1041
68	46 2400	46 3761	46 5124	46 6489	46 7856	46 9225	47 0596	47 1969	47 3344	47 4721
69	47 6100	47 7481	47 8864	48 0249	48 1636	48 3025	48 4416	48 5809	48 7204	48 8601
70	49 0000	49 1401	49 2804	49 4209	49 5616	49 7025	49 8436	49 9849	50 1264	50 2681
71	50 4100	50 5521	50 6944	50 8369	50 9796	51 1225	51 2656	51 4089	51 5524	51 6961
72	51 8400	51 9841	52 1284	52 2729	52 4176	52 5625	52 7076	52 8529	52 9984	53 1441
73	53 2900	53 4361	53 5824	53 7289	53 8756	54 0225	54 1696	54 3169	54 4644	54 6121
74	54 7600	54 9081	55 0564	55 2049	55 3536	55 5025	55 6516	55 8009	55 9504	56 1001
75	56 2500	56 4001	56 5504	56 7009	56 8516	57 0025	57 1536	57 3049	57 4564	57 6081
76	57 7600	57 9121	58 0644	58 2169	58 3696	58 5225	58 6756	58 8289	58 9824	59 1361
77	59 2900	59 4441	59 5984	59 7529	59 9076	60 0625	60 2176	60 3729	60 5284	60 6841
78	60 8400	60 9961	61 1524	61 3089	61 4656	61 6225	61 7796	61 9369	62 0944	62 2521
79	62 4100	62 5681	62 7264	62 8849	63 0436	63 2025	63 3616	63 5209	63 6804	63 8401
80	64 0000	64 1601	64 3204	64 4809	64 6416	64 8025	64 9636	65 1249	65 2864	65 4481
81	65 6100	65 7721	65 9344	66 0969	66 2596	66 4225	66 5856	66 7489	66 9124	67 0761
82	67 2400	67 4041	67 5684	67 7329	67 8976	68 0625	68 2276	68 3929	68 5584	68 7241
83	68 8900	69 0561	69 2224	69 3889	69 5556	69 7225	69 8896	70 0569	70 2244	70 3921
84	70 5600	70 7281	70 8964	71 0649	71 2336	71 4025	71 5716	71 7409	71 9104	72 0801
85	72 2500	72 4201	72 5904	72 7609	72 9316	73 1025	73 2736	73 4449	73 6164	73 7881
86	73 9600	74 1321	74 3044	74 4769	74 6496	74 8225	74 9956	75 1689	75 3424	75 5161
87	75 6900	75 8641	76 0384	76 2129	76 3876	76 5625	76 7376	76 9129	77 0884	77 2641
88	77 4400	77 6161	77 7924	77 9689	78 1456	78 3225	78 4996	78 6769	78 8544	79 0321
89	79 2100	79 3881	79 5664	79 7449	79 9236	80 1025	80 2816	80 4609	80 6404	80 8201
90	81 0000	81 1801	81 3604	81 5409	81 7216	81 9025	82 0836	82 2649	82 4464	82 6281
91	82 8100	82 9921	83 1744	83 3569	83 5396	83 7225	83 9056	84 0889	84 2724	84 4561
92	84 6400	84 8241	85 0084	85 1929	85 3776	85 5625	85 7476	85 9329	86 1184	86 3041
93	86 4900	86 6761	86 8624	87 0489	87 2356	87 4225	87 6096	87 7969	87 9844	88 1721
94	88 3600	88 5481	88 7364	88 9249	89 1136	89 3025	89 4916	89 6809	89 8704	90 0601
95	90 2500	90 4401	90 6304	90 8209	91 0116	91 2025	91 3936	91 5849	91 7764	91 9681
96	92 1600	92 3521	92 5444	92 7369	92 9296	93 1225	93 3156	93 5089	93 7024	93 8961
97	94 0900	94 2841	94 4784	94 6729	94 8676	95 0625	95 2576	95 4529	95 6484	95 8441
98	96 0400	96 2361	96 4324	96 6289	96 8256	97 0225	97 2196	97 4169	97 6144	97 8121
99	98 0100	98 2081	98 4064	98 6049	98 8036	99 0025	99 2016	99 4009	99 6004	99 8001
100	0	1	2	3	4	5	6	7	8	9

Table IV.—Natural Sines*

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0	0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	3907	3923	3039	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	10	12
36	5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

* From "Experiments in Physics," Ingersoll and Martin.

TABLE IV.—NATURAL SINES—*Continued*

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
45°	7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	4	6
67	9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	4	4
73	9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	1
85	9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	9994	9995	9995	9996	9996	9997	9997	9997	9998	9988	0	0	0	0	0
89	9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0
						nearly	nearly	nearly	nearly	nearly					

Table V.—Natural Cosines

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	1.000	1.000	1.000	1.000	1.000	1.000	9999	9999	9999	9999	0	0	0	0	0
1	9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	0	1	1	2
6	9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	4	4
18	9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	9205	9198	9191	9184	9178	9171	9164	9157	9150	9143	1	2	3	5	6
24	9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	9063	9056	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	8660	8652	8643	8634	8625	8616	8607	8599	8590	8581	1	3	4	6	7
31	8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	8290	8281	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	8090	8080	8070	8059	8049	8039	8028	8018	8007	7997	2	3	5	7	9
37	7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	7547	7536	7524	7513	7501	7490	7478	7466	7455	7443	2	4	6	8	10
42	7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

Note.—Numbers in difference columns to be subtracted, not added.

Table V.—Natural Cosines—*Continued*

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
45°	7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	6691	6678	6655	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	2924	2907	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	2756	2740	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	12	14
80	1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	1392	1374	1357	1340	1323	1305	1288	1271	1253	1236	3	6	9	12	14
83	1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	0523	0506	0488	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15

Note.—Numbers in difference columns to be subtracted, not added.

TABLE VI.—NATURAL TANGENTS*

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
0°	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	14
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	.1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3172	.3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	6	10	13	17
20	.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	10	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	10	14	19	24
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

* From "Experiment in Physics," Ingersoll and Martin.

TABLE VI.—NATURAL TANGENTS—*Continued*

	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	1	2	3	4	5
45°	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	26	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	23	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	17	25	33	41
54	1.3765	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6440	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	0057	0145	0233	0323	0413	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	74	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	118
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	130
71	2.9042	9208	9375	9544	9714	9887	0061	0237	0415	0595	29	58	87	115	144
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	82	122	162	203
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	94	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	214	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	62	124	186	248	310
78	4.7046	7453	7867	8288	8716	9152	9594	0045	0504	0970	73	146	219	292	365
79	5.1446	1929	2422	2924	3435	3955	4486	5026	5578	6140	87	175	262	350	437
80	5.6713	7297	7894	8502	9124	9758	0405	1066	1742	2432	Difference col-				
81	6.3138	3859	4596	5350	6122	6912	7720	8548	9395	0264	umns cease to be				
82	7.1154	2066	3002	3962	4947	5958	6996	8062	9158	0285	useful owing to the				
83	8.1443	2636	3863	5126	6427	7769	9152	0579	2052	3572	rapidity with which				
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95	the value of the				
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27	tangent changes.				
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	66.63	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					

INDEX